**PHYS 622**

**Problem set # 1 (due January 29)**

Each problem is 10 points.

**Q1** Consider an electron bound in a hydrogen atom under the influence of a homogeneous magnetic field \( \vec{B} = zB \). Ignore electron spin. The Hamiltonian of the system is:
\[ \hat{H} = \hat{H}_0 - \omega \hat{L}_z, \]
where \( \omega = |e|B/2\mu_c \). The eigenstates \(|n\ell m\rangle\) and eigenvalues \(E_n^{(0)}\) of the unerterbed hydrogen Hamiltonian \(\hat{H}_0\) are to be considered known. Assume that initially (at \(t = 0\)) the system is in the state:
\[ |\psi(0)\rangle = (|21 - 1\rangle - |21 1\rangle)/\sqrt{2}. \]

For each of the following state, calculate the probability of finding the system, at some later time \(t > 0\), in that state:
\[ |2p_1\rangle = (|21 - 1\rangle - |21 1\rangle)/\sqrt{2}; \]
\[ |2p_0\rangle = (|21 - 1\rangle + |21 1\rangle)/\sqrt{2}; \]
\[ |2p_2\rangle = |21 0\rangle. \]

When does each probability become equal to 1?

**Q2** For a spin-1/2 particle the most general form of the spin wave function is:
\[ |\psi\rangle = \begin{pmatrix} \cos \theta \\ e^{i\phi} \sin \theta \end{pmatrix}, \]
where \(0 \leq \theta \leq \pi/2\) and \(0 \leq \phi < 2\pi\). Find the direction in space \(\hat{n}\), such that this state corresponds to the \(+\hbar/2\) eigenvalue of the spin operator projection on this direction \(\hat{n} \cdot \hat{S}\). Use your solution to find the eigenvectors for the \(x\) and \(y\) components of the spin operators \(\hat{S}_x\) and \(\hat{S}_y\).

**Q3** Consider a spinless particle of mass \(\mu\) and charge \(q\) under the simultaneous influence of a uniform magnetic field and electric fields. The interaction Hamiltonian of the the two terms are:
\[ \hat{H}_I = -\frac{q}{2\mu c} \vec{B} \cdot \vec{E}; \quad \hat{H}_E = -qE \cdot \vec{r}. \]

Show that
\[ |\ell m\rangle \hat{H}_M + \hat{H}_E |\ell' m'\rangle|^2 = |\langle \ell m|\hat{H}_M|\ell' m'\rangle|^2 + |\langle \ell m|\hat{H}_E|\ell' m'\rangle|^2, \]
and that, always, one of the matrix elements \(\langle \ell m|\hat{H}_M|\ell' m'\rangle\) or \(\langle \ell m|\hat{H}_E|\ell' m'\rangle\) vanishes.

**Q4** Consider a pair of particles with opposite electric charges that have a magnetic-dipole-moment interaction
\[ \hat{H}_I = \alpha (\vec{p}_1 \cdot \vec{p}_2) = -\frac{e^2 g_1 g_2}{2m_1 m_2} \alpha \left( \vec{S}_1^{(1)} \cdot \vec{S}_2^{(2)} \right). \]

The system is subject to an external uniform magnetic field \(\vec{B}\), which introduces the interaction:
\[ -\vec{B} \cdot (\vec{p}_1 + \vec{p}_2) = -\frac{e}{2m_1 m_2} \vec{B} \cdot \left( m_2 g_1 \vec{S}_1^{(1)} - m_1 g_2 \vec{S}_2^{(2)} \right). \]

Determine the energy eigenvalues and eigenstates (in the basis of the spin eigenstates \(|\uparrow\rangle^{(1)}|\downarrow\rangle^{(2)}|\uparrow\rangle^{(1)}|\downarrow\rangle^{(2)}\)). Express the results in terms of the parameters \(\alpha = (e^2 g_1 g_2)/4m_1 m_2\) and \(b_i = g_i B_i/4m_i\).

**Q5** Consider the state \(|j_1, j_2; j m\rangle\), which is a common eigenstate of the angular momentum operators \(\vec{J}_1^2\) and \(\vec{J}_2^2\), where \(\vec{J} = \vec{J}_1 + \vec{J}_2\). Show that this state is also an eigenstate of the inner product operator \(\vec{J}_1 \cdot \vec{J}_2\) and find its eigenvalues. Do the same for the operators \(\vec{J}_1 \cdot \vec{J}\) and \(\vec{J}_2 \cdot \vec{J}\).