1. **Vacuum Solutions in Three Dimensions**

In this problem we will look for static, isotropic black-holes in three space-
time dimensions.

Assume a metric of the form

\[ ds^2 = -e^{2\phi(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2d\theta^2. \]

a) Calculate the nonvanishing Christoffel symbols.

b) Calculate the components of the Ricci tensor \( R_{tt}, R_{rr}, \) and \( R_{\theta\theta}. \) The other components of \( R_{\mu\nu} \) vanish.

c) Solve the vacuum Einstein equations \( R_{\mu\nu} = 0, \) assuming that the metric approaches the flat metric as \( r \to \infty. \) Are there any nontrivial black-hole solutions?

2. **Newtonian Stars**

Consider static, spherically symmetric solutions to Einstein’s equations for
a fluid with density \( \rho(r) \) and pressure \( p(r), \) with metric of the form,

\[ ds^2 = -e^{2\phi(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2). \]

Assume the fluid is nonrelativistic, and consider the nonrelativistic limit
of Einstein’s equations for this system. Assume \( \phi(0) = \lambda(0) = 0, \) and

\[ \rho(r) = \rho_0 \left(1 - \frac{r}{R}\right) \]

for \( r \leq R, \) and \( \rho(r) = 0 \) for \( r > R. \)

a) Find the spacetime metric for \( r < R \) and \( r > R. \)

b) Find the pressure in the star \( p(r), \) such that \( p(R) = 0. \)
3. *Binary Star Systems*

Assume the two stars in a binary star system each have mass \( M = 1.39 \) solar masses, with orbital period \( T = 7.75 \) hrs, and semimajor axis with respect to one of the stars \( a = 2R = 1.95 \times 10^6 \) km.

a) Assuming circular orbits find the power radiated in gravitational radiation in Watts.

b) In the same system, find the change in the orbital period after each complete orbit.