1. **Death by Black Hole**

Suppose a two-meter-tall human being falls feet-first into a black hole with the mass of the sun. Suppose the human can withstand the tidal acceleration gradient until the feet would accelerate 1000 m/s\(^2\) more than the head along a geodesic. What value of \(r\) in standard coordinates do the feet reach before the human dies?

*Hint:* the tidal acceleration gradient is determined from the geodesic deviation \(\frac{D^2}{D\tau^2}(\delta x^\mu)\).

2. **Encircling the Universe**

Show that a photon emitted at the big bang travels once around the universe in a matter-dominated \(k = +1\) FRW universe (with vanishing cosmological constant).

3. **FRW Universe with Cosmological Constant**

a) Beginning with the FRW form of the metric and assuming the energy-momentum tensor of a perfect fluid, show that if Einstein’s equations are modified to include a cosmological constant:

\[
R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \Lambda g_{\mu\nu} = -8\pi G_N T_{\mu\nu},
\]

then the scale factor of the universe satisfies

\[
\dot{R}^2 + k = \frac{8\pi G_N}{3} \rho R^2 + \frac{\Lambda}{3} R^2.
\]

b) Show that if \(\Lambda\) is large enough a \(k = 1\) universe can expand forever.