Problem 1. [25 points]

A single ice cube has a mass of 20 grams. It is initially immersed in liquid nitrogen in order to bring its temperature to -195°C. It is then placed in one half liter of water (mass of 500 grams) which is initially at room temperature, 20°C, and is in an insulated container. What is the final temperature of the system? Hint: all the ice will melt.

Possibly useful constants: $c_{ice} = 0.50 \text{ kcal/kg°C}$, $c_{water} = 1.00 \text{ kcal/kg°C}$,
$L_{f_{water}} = 80 \text{ kcal/kg}$, $L_{f_{water}} = 539 \text{ kcal/kg}$.

Calorimetry Problem: $Q_{heat} + Q_{add} = 0$

$Q_{heat} = Q_1 + Q_2 + Q_3$ \[ Q_1 = \text{raise ice to melting point} = m_i C_i(0°C - (-195°C)) \]

$Q_2 = \text{melt ice} = m_i L_f$ \[ Q_3 = m_i C_o(T - 0°C) \]

$Q_{add} = m_w C_o(T - 20°C)$

$Q_{heat} + Q_{add} = 0$ \[ m_i C_i(195°C) + m_i L_f + m_w C_o T + m_w C_o(T - 20°C) = 0 \]

$T(m_i C_o + m_w C_o) = m_w C_o(20°C) - m_i C_i(195°C) - m_i L_f$

$T = \frac{m_w C_o(20°C) - m_i C_i(195°C) - m_i L_f}{m_i C_o + m_w C_o}$

$= \frac{(0.5kg)(10 \text{ kcal/kg°C})(20°C) - (2 \times 10^{-3} \text{ kg})(0.5 \text{ kcal/kg°C})(195°C) - (2 \times 10^{-3} \text{ kg})(80 \text{ kcal/kg})}{(2 \times 10^{-3} \text{ kg})(10 \text{ kcal/kg°C}) + (0.5 kg)(10 \text{ kcal/kg°C})}$

$= 12.4°C$

(see lecture notes from Jan 27th)
& problem #3 on Homework 2
Problem 2. [20 points]

Two moles of an ideal diatomic gas expands from 0.01 m$^3$ to 0.03 m$^3$ at a constant pressure of 800 kilopascals.

a) [5 points] Sketch the process on a P-V diagram.

b) [7 points] How much work is done by the gas during this expansion?

c) [8 points] How much heat flows into the gas during this expansion?

\[
\begin{align*}
\text{P-V graph} &:\text{Isobaric Process} \quad \text{V} = 8 \times 10^5 \text{ Pa} \\
\Delta U &= Q - W \\
\frac{\Delta U}{\frac{nRT}{2}} &= \frac{5}{2} \text{P} \Delta V \\
Q &= \Delta U + W = \frac{5}{2} \text{P} \Delta V + W = \frac{7}{2} \text{P} \Delta V = \frac{7}{2} (1.6 \times 10^4 \text{J}) = 5.6 \times 10^4 \text{J}
\end{align*}
\]

Problem 3. [15 points]

A heat engine operates using the Carnot cycle. Every second, it produces 200 J of work, while dumping 500 J of waste heat into a cold reservoir, which is at room temperature (20°C).

a) [10 points] What is the temperature of the hot reservoir?

b) [5 points] What would be the coefficient of performance of a Carnot-cycle refrigerator operating between these two temperatures?

\[
\begin{align*}
\text{Th} &\rightarrow Q_h \\
\text{Tc} &\rightarrow Q_c \\
Q_h &= Q_c + W = 200 \text{J} + 500 \text{J} = 700 \text{J} \\
\epsilon &= \frac{W}{Q_h} = 1 - \frac{T_c}{T_h} \\
&= \frac{200}{700} = \frac{2}{7} \\
&\therefore \frac{T_c}{T_h} = 1 - \frac{2}{7} = \frac{5}{7} \\
&\therefore T_h = \frac{7}{5} T_c \\
&= \frac{7}{5} (298 \text{K}) = 410 \text{K} \\
\text{COP} = \frac{Q_c}{W} = \frac{500 \text{J}}{200 \text{J}} = 2.5
\end{align*}
\]
Problem 4. [15 points]

A 300.0 cm long piece of an unknown metal, which is at room temperature (20°C) is given to you. You suspect that it is either made of steel or aluminum. You heat it up to 100°C and find that the length of the piece is now 300.6 cm. The coefficient of linear expansion for steel is $25 \times 10^{-6} \text{C}^{-1}$ and the coefficient for aluminum is $12 \times 10^{-6} \text{C}^{-1}$.

Is the piece made of steel or aluminum? Show your calculation.

$$\Delta L = \alpha \Delta T$$

$$\alpha = \frac{\Delta L}{\Delta T} = \frac{(300.6 - 300.0) \text{cm}}{(300 \text{ cm})(100 \text{C} - 20 \text{C})} = 2.5 \times 10^{-5} \text{C}^{-1} \quad \therefore \text{steel}$$

(See, e.g., problem 23 on Homework #1)

Problem 5. [15 points]

In one minute, how much heat is transferred by radiation out of a car radiator at 110°C into a 50°C environment? The radiator has an emissivity of 0.750 and a 1.20 m² surface area.

$$\frac{Q}{t} = \sigma \varepsilon A (T^4_{\text{hot}} - T^4_{\text{cold}})$$

$$T_{\text{hot}} = 110^\circ \text{C} = 383 \text{K}$$

$$T_{\text{cold}} = 50^\circ \text{C} = 323 \text{K}$$

$$\frac{Q}{t} = \left(5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \cdot \text{K}^4}\right) (0.750) (1.2 \text{m}^2) \left[(383 \text{K})^4 - (323 \text{K})^4\right]$$

$$= 544 \text{W}$$

$$Q = \frac{Q}{t} \cdot t = (544 \text{W})(60 \text{s}) = \frac{(544 \text{J})(60 \text{s})}{5} = 32.7 \text{kJ}$$

(See lecture notes; example done Jan. 29)
Problem 6. [10 points]

Suppose that the average velocity \( v_{\text{rms}} \) of carbon dioxide molecules (molecular mass 44.0 grams per mole) in a flame is found to be \( 1.00 \times 10^5 \) m/s.

a) [5 points] What temperature does this represent?

\[ U_{\text{rms}} = \left( \frac{3kT}{m} \right)^{\frac{1}{2}} \quad \Rightarrow \quad U_{\text{rms}}^2 = \frac{3kT}{m} \]

\[ \therefore T = \frac{mU_{\text{rms}}^2}{3k} = \frac{(4.4 \times 10^{-3} \text{ kg/mol}) (1.00 \times 10^5 \text{ m/s})}{3 (1.38 \times 10^{-23} \text{ J/K})} \]

\[ = \boxed{1.77 \times 10^7 \text{ K}} \quad \text{Wow! Hot stuff!} \]

(see Problem #10 in Homework #1)

b) [5 points] If the average velocity were, instead, 1/4 of this value, what temperature would this now represent?

\[ T \propto (U_{\text{rms}})^2 \]

\[ \therefore \quad \left( \frac{1}{4} \right)^2 = \frac{1}{16} \quad \Rightarrow \quad T_{\text{new}} = \frac{1}{16} T = \boxed{1.11 \times 10^6 \text{ K}} \quad \text{still rather hot!} \]