Symbolic analysis of non-stationary time series

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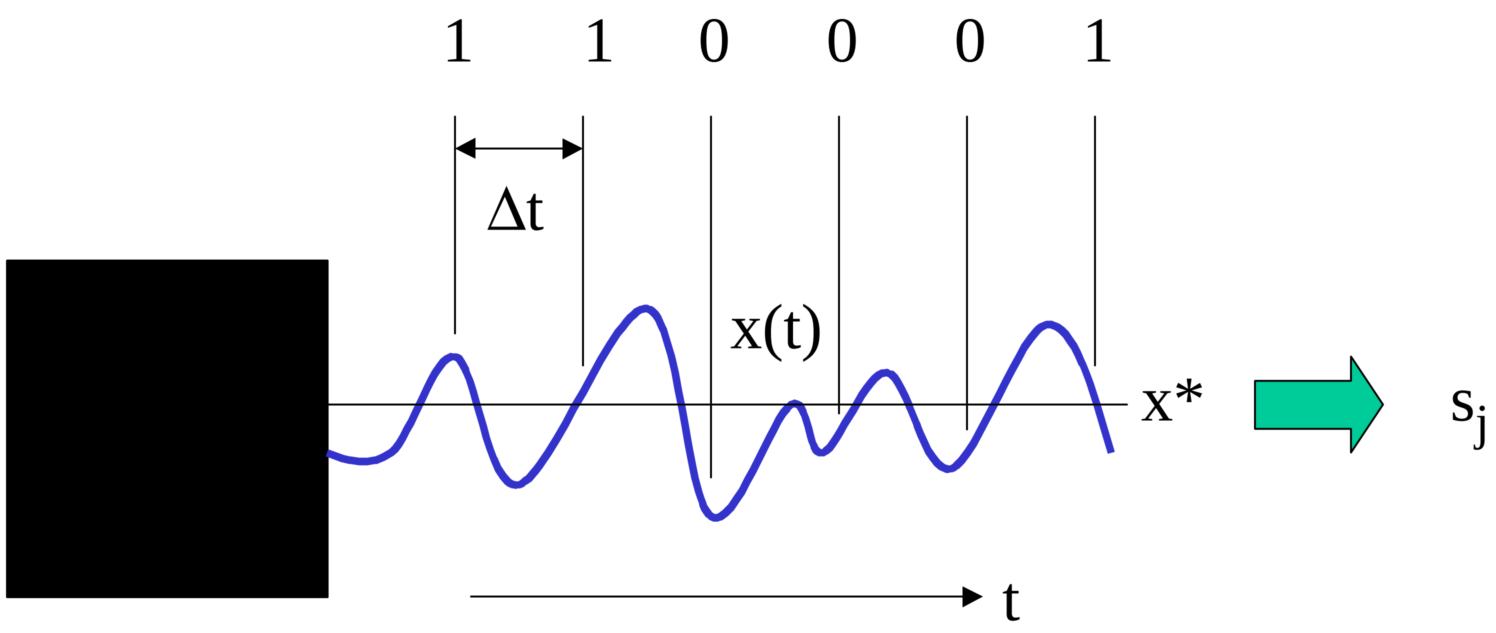
Goals

- Detection and characterization of non-stationarity from observations.
- Early warning of impending instabilities from noisy and/or short time series.
- Improved control/stabilization of instabilities.

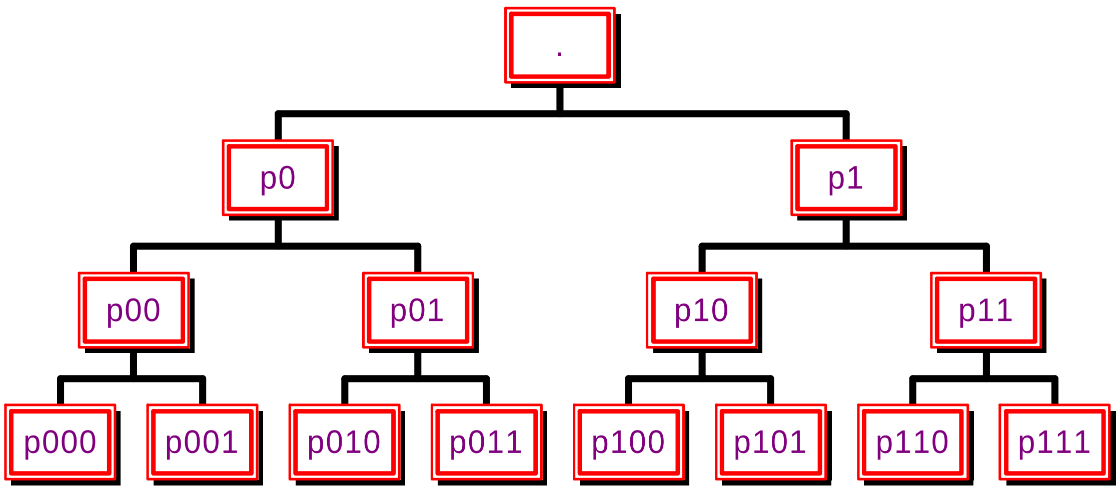
Symbolic time series analysis



S = 0111010101110101...

Now study statistical properties of the symbol string...

The symbol tree



Level	No. of symbols
1	2
2	4
3	8
L	2^L

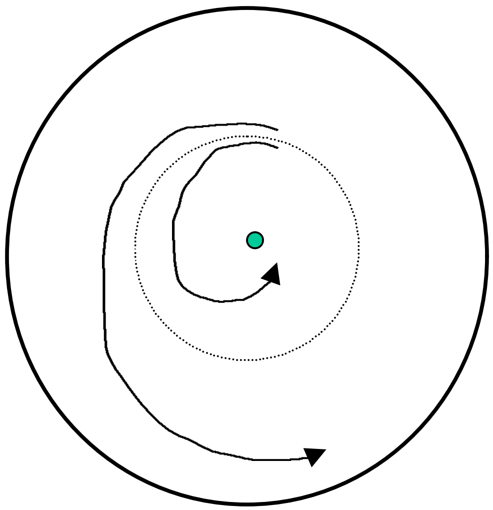
Prior work *= incl. application to non-stationary signals

- Characterization of complex signals (Crutchfield, PRL)*
- Modeling and parameter estimation (the measurement problem) (Tang & Tracy, PRE)
- Estimation of timescales/detection of weak periodicities (Tang & Tracy, Chaos)
- Detection/control of period doubling bifurcations in internal combustion engines, classification of dynamics of fluidized bed reactors (Daw et al., PRE)*
- Construction of finite-state models, detection of dynamical correlations (Rechester & White, Phys. Lett., PRL)
- Characterization of heart signals, astrophysical signals (Kurths, et al., Chaos, PRE)*
- Detection of non-stationarity (Burton & Tracy, unpub.)

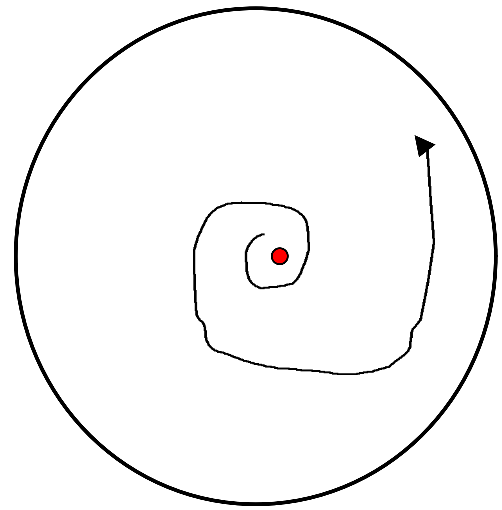
Key issue for non-stationary time series: small sample effects

- Find *shortest* window length which still gives statistically significant estimate of the symbol tree at level L.
- Bootstrap confidence level estimate of variability of symbol statistics in order to detect *statistically significant* changes.
- Characterize changes to isolate *dynamically significant* changes (e.g. instability precursors).

Benchmark problem: early warning of sub-critical Hopf bifurcations



$\lambda<\lambda_c$



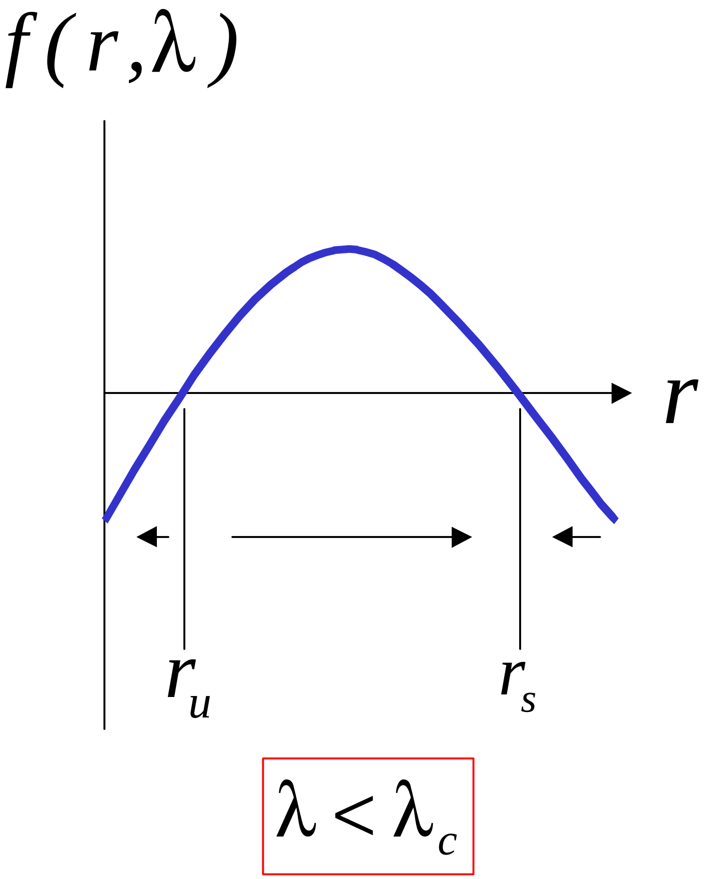
$\lambda>\lambda_c$

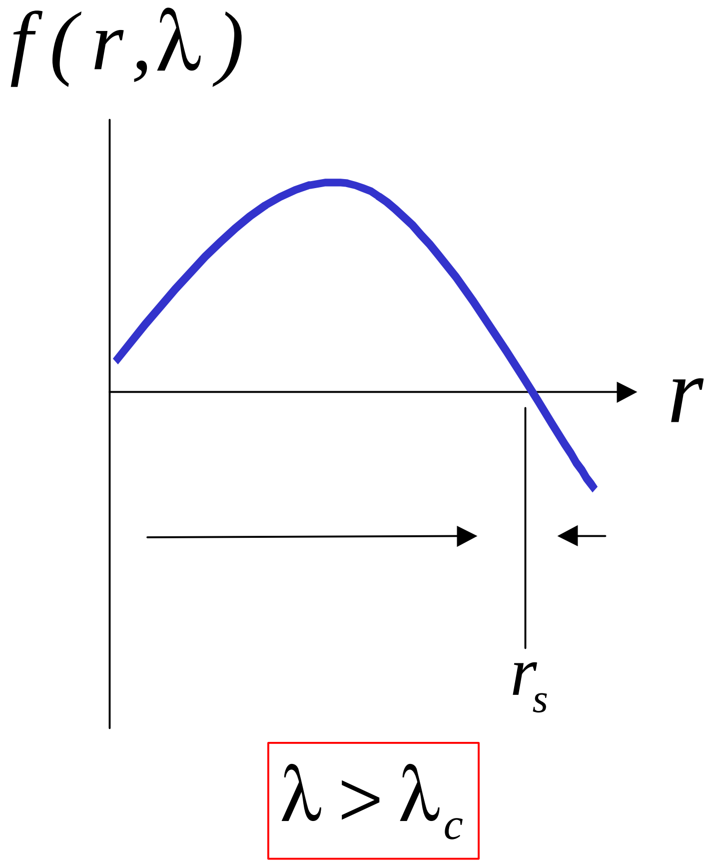
For $\lambda = \lambda(\varepsilon t)$ then loss of stability occurs at a time t_c. With noise driving, the system loses stability at some $t<t_c$.

Key: — stable limit cycle; ········· unstable limit cycle

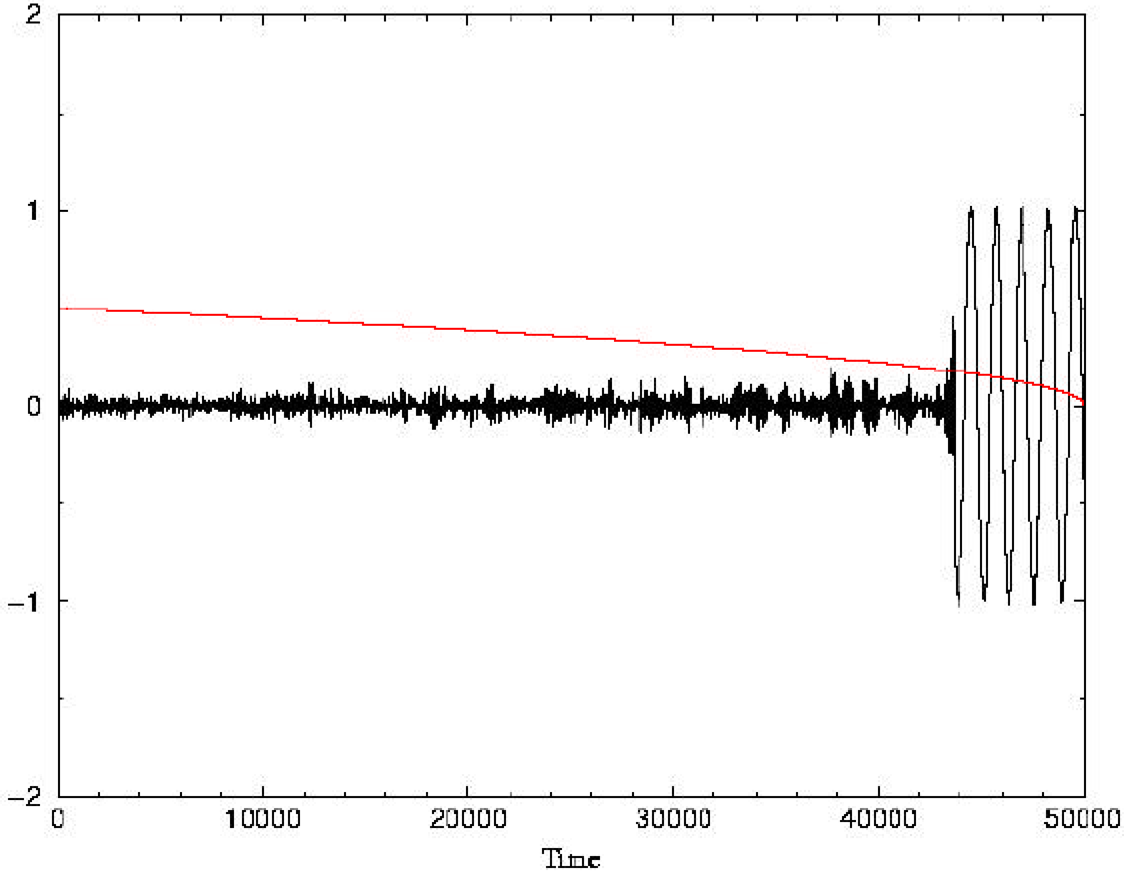
● stable fixed point ● unstable fixed point

$$\dot{r} = f(r, \lambda(\varepsilon t)) + \eta(t), \qquad \dot{\theta} = \omega - \gamma r^2 + \xi(t)$$



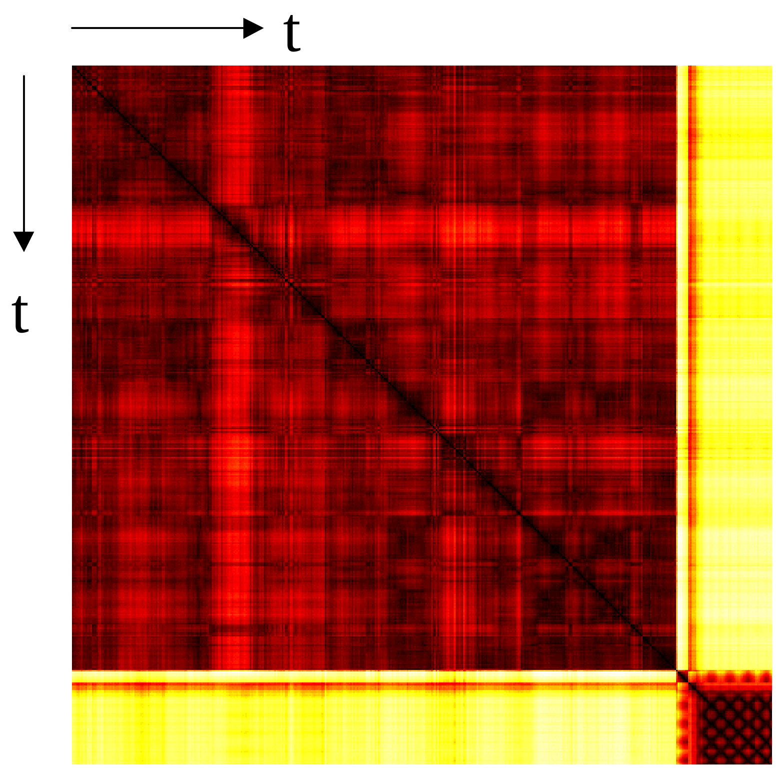


Subcritical Hopf instability

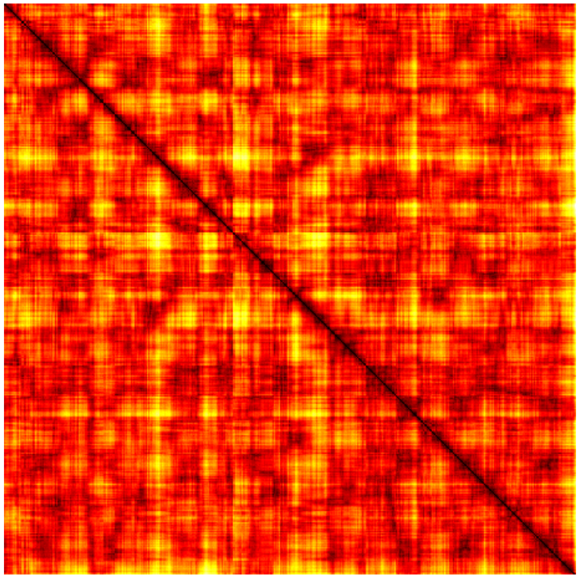


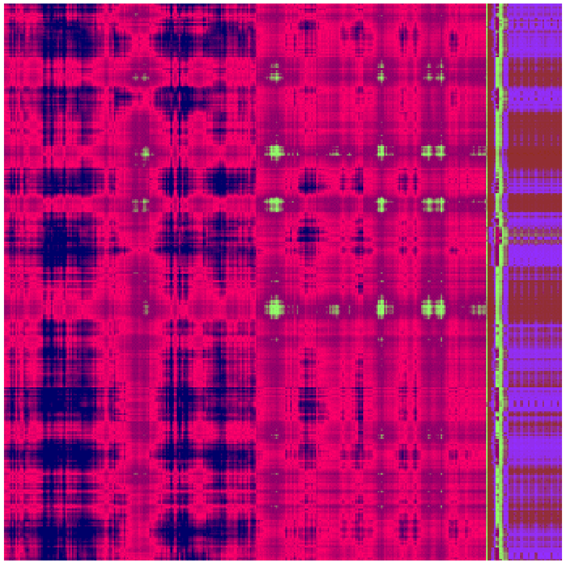
Search for changes in fluctuations:

- Sample symbol stats w/moving 100 pt. windows
- Tree level 4
- Take all pairwise diffs.
- Plot landscape
- Characterize *significant* variability (Statistical? Dynamical?)



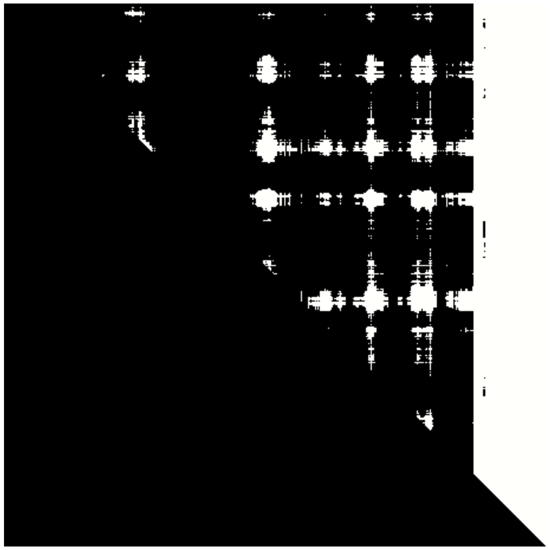
Drive-drive
(ran1 from
Num. Rec.)

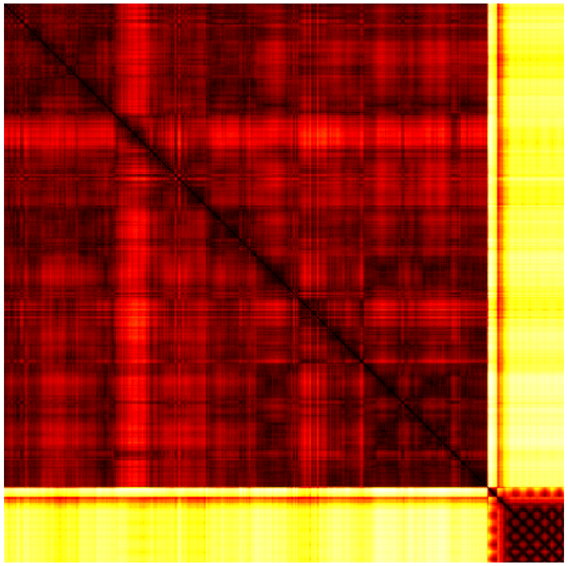




Drive-resp.

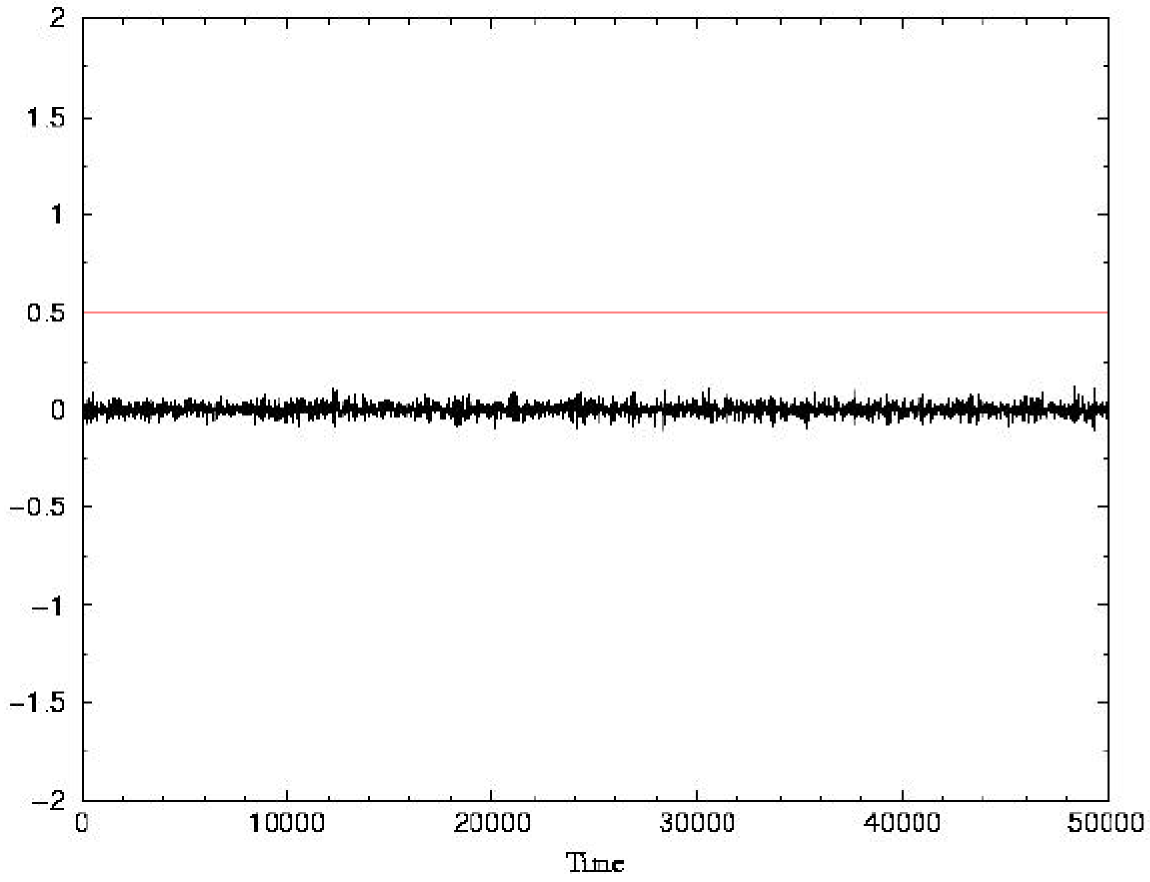
Drive-response
> 95%





Resp.-resp.

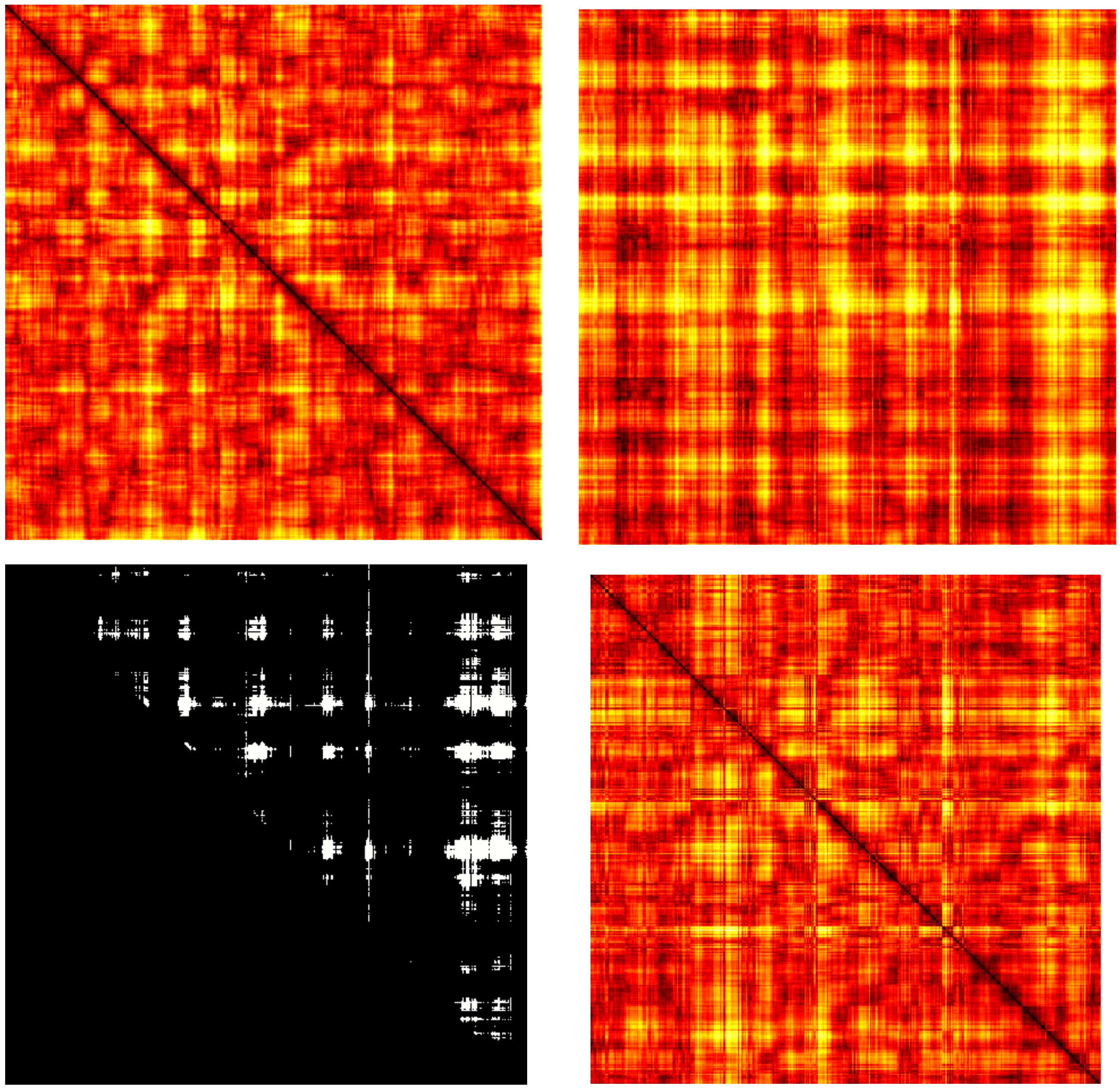
Null test: λ held fixed (no instability)



Null test: no instability

Drive-drive

Drive-resp.



Drive-response
> 95%

Resp-resp.

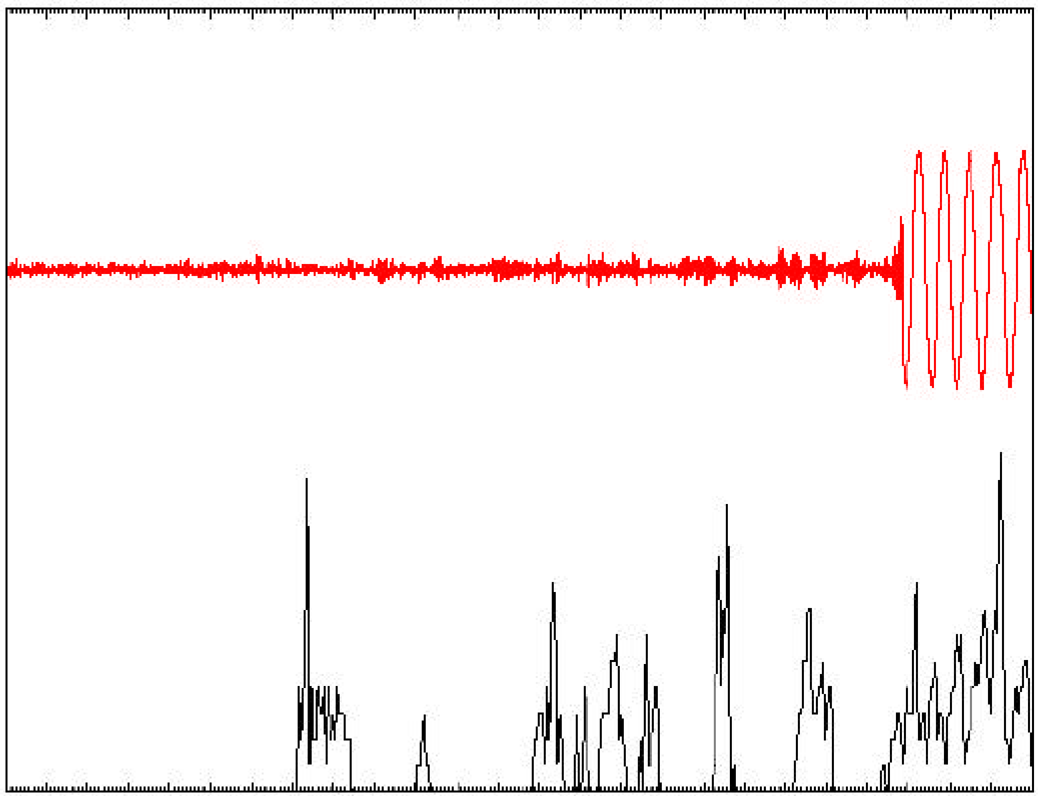
Sound the alarm!

- Early as possible
- with as few false alarms as possible
- set alarm threshold such that: triggered by drive-response outliers + low drive variability.

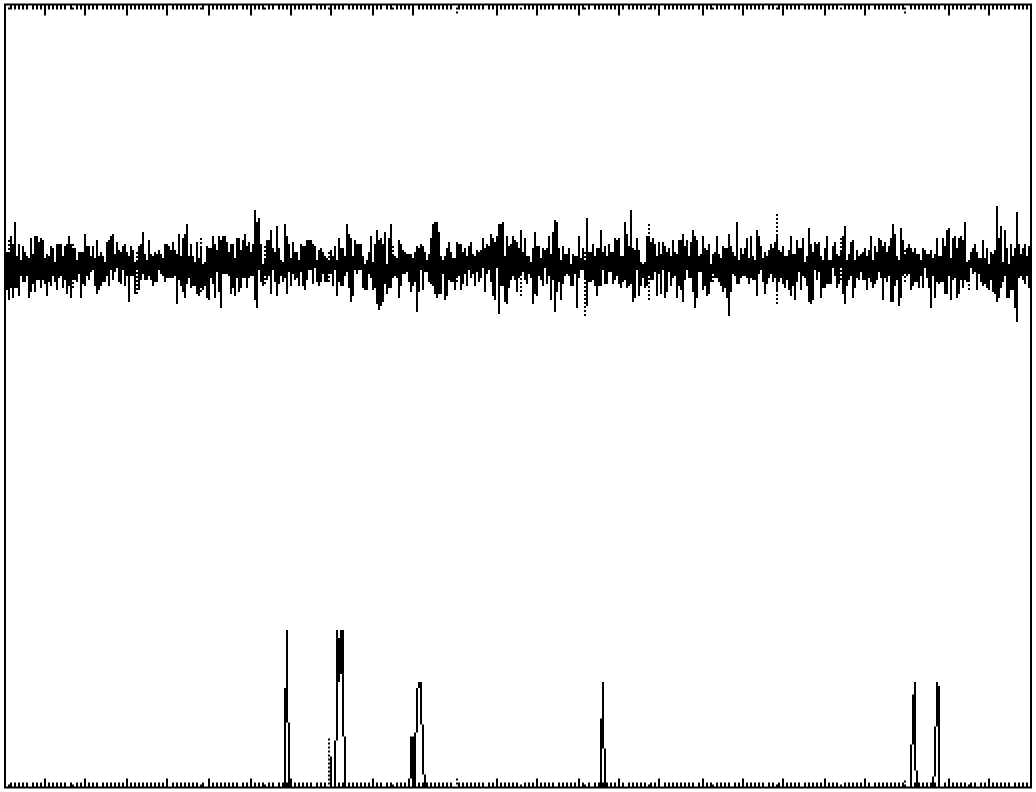
$$P(D-R) > 95\% \quad and \quad P(D-D) < T\%$$

- This selects patterns that show dynamically significant shift in drive-resp. character.

Alarm vs. time for Hopf bifurcation



Alarm vs. time for null test



Conclusions:

- Symbol statistics are very sensitive to non-stationarity in time series (they can *easily* detect non-stationarity in random number generators, Burton & Tracy, unpublished)
- Cross-comparisons of drive variability with response variability shows promise for detection of significant changes in drive-response characteristics.

Conclusions:

- Future work will also consider classification.
- Long-term goals include development of symbolic controllers (see, e.g. Daw) to ameliorate or stabilize instabilities. Requires faster response/smaller windows.