(2) Consider a one-dimensional simple Harmonic oscillator. Let $|n\rangle$ denote its n-th normalized eigenstate ($n = 0, 1, \cdots$).

1. Evaluate $\langle n'|x|n\rangle$, $\langle n'|x^2|n\rangle$, $\langle n'|p|n\rangle$, and $\langle n'|p^2|n\rangle$ for any n and n'.
2. Suppose the system is in the state $|\psi\rangle = a|0\rangle + b|1\rangle$, where a and b are real numbers. Show that the expectation $\langle x\rangle = \langle\psi|x|\psi\rangle$ is in general not equal to zero. What values of a and b would give the maximum value of $\langle x\rangle$? Sketch the wave function $\psi(x) = \langle x|\psi\rangle$ for this choice of a and b, together with the eigenfunctions $\langle x|0\rangle$ and $\langle x|1\rangle$. For the last part (the sketch), set $\hbar = m = \omega = 1$.
3. Suppose the system is in $|0\rangle$. Now suddenly the origin (i.e., the point to which the "spring" is attached) is displaced from 0 to x_0. The wave function does not change during the displacement. What is the probability that we will find the oscillator in the new ground state, i.e., the ground state with respect to the new origin?