(2) (*From Sakurai Problem 3.2.*) Consider the 2×2 matrix defined by

$$U = \frac{a_0 + i\sigma \cdot \mathbf{a}}{a_0 - i\sigma \cdot \mathbf{a}},$$

where σ denotes the Pauli matrices, a_0 is a real number, and $\mathbf{a}$ is a three-dimensional vector with real components. (The notation $\frac{A}{B}$ denotes AB^{-1} for two square matrices A and B.)

1. Prove that U is unitary and unimodular (i.e., $\det(U) = 1$.)

2. In general, a 2×2 unitary unimodular matrix represents a rotation in three dimensions. That is, it has the form of $\exp[-i\mathbf{S}\phi/\hbar]$, where $\mathbf{S} = (\hbar/2)\, \sigma \cdot \hat{\mathbf{n}}$ is the general spin operator for a spin-1/2 system and ϕ is the rotation angle. Find $\hat{\mathbf{n}}$ and ϕ for U in terms of a_0, a_1, a_2, and a_3.