Number and System

THE TRANSCENDENCE OF NUMBER

Pythagoras claimed that, in some sense, All is Number; Aristotle believed this to be absurd.

Numbers began simply as adjectives, and then became more than a specie of words. Pythagoras claimed they were everything. He left us an enigmatic credo—reportedly something like *All is Number*. His was an intuition that had been inspired by ancient arts of measurement, expressed in numerology, and recently certified by a discovery of the relation between integer ratios and musical harmony. Numbers had become more than names for quantities; they measured the sky, the earth, and, through music, touched the soul.

In his *Metaphysics* Aristotle states:

> *On the other hand the Pythagoreans, because they see many qualities of numbers in bodies perceived by sense, regard objects as numbers,…And why? Because the qualities of numbers exist in a musical scale, in the heaven and in many other things. But for those who hold that number is mathematical only, it is impossible … to say any such thing, … they construct out of numbers physical bodies—out of numbers that have neither weight nor lightness, bodies that have weight and lightness—they seem to be speaking about another heaven and other bodies than those perceived by sense.*

> *The Pythagoreans … construct the whole heaven out of numbers, but not out of abstract numbers, for they assume that the units have magnitude;*
>
> *…seem to consider that number is the principle both as matter for things, and as constituting their attributes and permanent states.*
>
> *…but it is impossible that bodies should consist of numbers, and that this number should be mathematical.*

The Pythagoreans, it seemed, thought that somehow number both *is* and *describes* matter; Aristotle, struggling with this idea, could not understand how this could be. That number completely describes matter is a credo of modern science, expressed by Leibniz as *God is a Mathematician,* and by Galileo's statement that *the book of nature is written in mathematical letters*. Most people already have difficulty believing that number completely describes matter, especially if they assume that they themselves are no more than a form of matter. As for the idea that number in some sense *is* matter, they would easily agree with Aristotle that it is absurd.

The mystic vision of Pythagoras exceeded that of Aristotle, the great pedestrian.

Aristotle is famous as a great classifier; he labeled things, defined them, put them in proper place and order. Where did he put number? What is its definition and place? The quotations above express concern about two senses of the word. In one sense, expressed by the phrase 'number as *mathematical*', number is a concept; in another sense, numbers constitute *matter for things*. These are quite different.

The Pythagoreans used number in both senses. They used number as a concept just as we do today, for example, to count; but they also thought of numbers as things—they *assume that the units have magnitude*. Numbers were conceived somewhat as atoms and structures corresponding to number. The most important of these, for example, was called the *tetrakys*. It corresponded to the number 10 and to the triangular structure shown. This structure determined properties of the physical object of which it was part much as crystalline structure is understood today to determine properties.

1
2 3
4 5 6
7 8 9 10

Details are not important; but the visionary spirit of the enterprise is. This is especially apparent in the purposeful blurring of the boundaries between the meanings of number that Aristotle testifies to and fusses over. The questioning of definitions implicit in this, is the precursor of the fragmentation of basic vocabulary that accompanies conceptual breakthroughs. And when this is expressed millennia in advance of the fact, it can only be done enclosed in a mystical aura.

The connection between number and matter expressed by the Pythagorean credo is can now more easily be seen to be close to the spirit of modern physics. Physicists have mathematicized nature so thoroughly that they now only think of matter in terms of the symbols of mathematics. They rarely think of matter in any way distinct from its mathematical representation, and the thought that mathematical structures representing forms of matter are not identical to the latter, has no actual effect on doing physics. If so, what can the distinction between the mathematical and the sensual representation of matter mean? Which is more ‘real’, the mathematical or the sensual, and what does such reality mean?

The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is still today seen as a mystical and wonderful gift.

In his essay entitled *The Unreasonable Effectiveness of Mathematics in the Natural Sciences*[2], Eugene Wigner, one of the great mathematical physicists of our time, tells of two former high school classmates talking about their jobs. One had become a statistician and was working on population trends. He showed his friend a paper with the formula for the bell shaped curve used in statistics, and indicated the mathematical symbols which designated properties of the population distribution it described—its mean, its width, and so on. His classmate was slightly incredulous and suspected that the statistician was pulling his leg. He pointed to something on the paper and asked,

...And what is this symbol here?

Oh, this is pi.

What is that?

The ratio of the circumference of a circle to its diameter.

Well, now you are pushing things too far," said the classmate, "surely population has nothing to do with the circumference of a circle .

End of story. The classmate could not understand why the very special ratio π=3.14159… expressing a geometric ratio should also pop up in formulas concerned with human population. For that indicates some relation between them, and why, indeed, should circles and populations be related? Reasonable people would agree with the classmate.

Wigner's listeners were not reasonable people—they were mathematicians. They recognized his punch line, so he was able to just continue as follows:

Naturally we are inclined to smile about the simplicity of the classmate's approach. Nevertheless, when I heard this story, I had to admit to an eerie feeling because, surely, the reaction of the classmate betrayed only plain common sense

The classmate's common sense reacted to the fact that,

...mathematical concepts turn up in entirely unexpected connections. Moreover they often permit an unexpectedly close and accurate description of the phenomena in these connections.

Wigner knew his audience so expected such unexpected connections that they would immediately smile at the naiveté of the classmate. Indeed, his goal was to remind them of how unreasonable and, as he himself put it, eerie, this situation really is:

The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve. We should be grateful for it and hope that it will remain valid in future research and that it will extend....perhaps also to our bafflement, to wide branches of learning.

Two things should separately surprise all of us: "The miracle of the appropriateness ... of mathematics ... ", and "[the] entirely unexpected connections [it turns up]...". In the story, the appropriateness of mathematics is perhaps not so surprising since statistics does, after all, involve numbers. But that anything might connect counting people to geometry—connect the structures of population and space—should amaze anyone.

Mathematics reveals many such amazing connections, each the product of numerous steps of perfect logic. Although each step--each elementary numerical or algebraic manipulation--is easy to perform and understand, a path made up of numerous steps quickly becomes too long to comprehend as a whole, and so the connection it creates can remain amazing forever. This is part of the answer to Wigner's story.

THE EMERGENCE OF NUMBER

Any system of thought and symbol that promotes the creation of numerous steps of perfect logic eventually becomes a form of mathematics. These systems of thought are mental machines with perfectly interlocking parts. The first were number systems, with numbers, their parts.

This section identifies and discusses, through the filter of history, the significance of number systems. In so doing it reveals some of the reasons for the truth of the Pythagorean dictum.

The use of number long preceded counting.

One of the oldest surviving archaeological records of number is a 30,000 year old Paleolithic tally stick found in Moravia. It is a wolf's bone, 7 inches long, engraved with 55 notches. From the same period comes a bone plaque found in France, also notched with what appears to be a record of the sequential phases of the moon for two and a quarter months. A later plaque from 11,000BCE has a more complex notation and covers 3 1/2 years of lunar-solar observation[4]. Their use did not involve ability to count, the procedure we use to identify numbers beyond a mere *handful*. We do not remember more than a handful of numbers, instead we have a *system* for sequentially constructing names as we tick off items (which is what counting means) and we know that such systems evolved far more recently than 11,000BCE.

To compare numbers of notches on two tally sticks, the Moravian made a ‘one-to-one correspondence’ between them. Each notch on one stick was matched with a notch on another stick. The stick with extra, unmatched notches designated the larger number. How much one number was greater than the other could not be answered except by pointing to another tally stick.

The systematization of number names is an early example of reduction.

When a society uses only a few objects at a time, only a few numbers are needed, and any set of names for them will do. Many numbers, however, would require many names which, if not systematically defined, would be useless. Imagine memorizing 100 systematically chosen names for the first 100 integers. If you can, try 1000.

Modern English contains the residue of unsystematic names up through twelve. Beyond that it became systematized. One might say that, in effect, a potentially infinite number of unsystematic names were reduced to an handful of numbers and rules. Our naming system allows us to easily count as high as we wish: we can count to 10000 as easily as to 100.

The systematization of number is an example of how mental labor saving inventions creates understanding and meaning.

The naming system has done more for us. Suppose someone mentions seventy-three objects. Since you cannot see seventy-three objects in your mind’s eye, you have no idea if this is a lot or a little except by virtue of the information contained in its name: it is one more than seventy-two, it is more than anything in the sixties, and so on. All these relations are conveyed merely by the name, by virtue of the naming rules, and done so rapidly, because the rules are so few and so easily remembered.

Without a system, you could not know relative sizes without bald memorization. For 100 numbers, remembering relative sizes would require remembering 5,000 relations of the form “X is greater than Y”. Thus, systematized naming reduces not only all the names, but all the most basic relations of number to a few simple rules. These relations constitute the *basic meaning* of number; you know what seventy-three means because you instantly have them mentally available.

Systematized naming was a labor saving device, invented to reduce mental work involved in the memorization, retrieval, and use of information. It was a mental invention, just as surely as a piece of technology is a physical invention, and one from which all of mathematics has evolved and upon which all modern science is based.

Written symbols were based on tokens; both were objects that abstracted data relevant to accounting.

The history of written numerical symbols is closely connected to that of writing in general, much of which has emerged[5] from a study of over 8,000 clay artifacts called tokens--small fired clay objects from 1-3 cm across--that have been found scattered throughout the Near East. They first appeared about 11,000 years ago, at about the same time as the first cultivation of cereals. This suggests that newly expanded economy, based on the discovery of agriculture at that time, generated their use. Each token represented one product—one animal of one type, one measure of grain, one jar of oil, and so on. Shape indicated type: ovoids represented jars of oil; spheres, measures of grain; disks, animals.

A token represents an abstraction. A sheep is (1) a discrete object, (2) distinguishable from objects of other types (jars, ewes,…), and (3) a member of a set of similar objects (the set of all sheep). These properties—being a discrete distinguishable member of a recognizable set--abstracted from all properties of sheep, are the only ones relevant for the intended use of the tokens; all others--properties of shape, skin, smell and so forth—are discarded as irrelevant to accounting purposes.

We will see that our sensory system unconsciously abstracts in this way continually (conscious mental abstraction, of course, is well known). Abstraction is a component of data reduction, built into the sensory system for the purpose of discarding irrelevant incoming data.

Written symbols were abstractions of tokens.

The earliest tokens are found at Uruk, the first and foremost Sumerian city on the ancient delta of the Euphrates, situated between Ur of the Chaldees and Babylon, in southern Mesopotamia. These were plain tokens, found in 16 unmarked shapes. Complex, or marked, tokens began appearing after about 4 millennia. The changeover came with the first efflorescence of manufacture; the greatly expanded range of products the tokens could represent extended from textiles and garments to bread and trussed ducks.

Marking was a labor saving device that facilitated a dramatic increase in economic complexity, by providing an easy way to create hundreds of types of complex tokens. The increase in token complexity continued for about another 500 years, until about 5600 years ago. During this period, people still required a single solid object—a token—to represent another single solid object. They were not prepared to utilize the fact that this solidity was irrelevant to record keeping and accounting.

The abstraction of information from its solid forms

Like earlier ones, the next step of abstraction on the road to writing, discarding of solidity of token, was also generated by economics. Clay envelopes containing tokens served as bills of lading accompanying shipments of goods. Originally, they had to be broken open to see their contents, but about 5600 years ago Sumerian accountants began pressing tokens onto the outer wet clay surface of the envelope before filling, sealing, and baking them. The external impressions made it unnecessary to break open the envelope to know what was inside; once the irrelevance of what was inside became obvious, writing began its birth. It took yet another five centuries for first the envelopes and then tokens themselves to disappear.

Analysis that separates mixed concepts and their symbols into their pure components is a critical form of abstraction.

By 5100 years ago, pictographic script traced with a stylus on a clay tablet came into its own. This then allowed the next abstraction, that of quantity from single incised symbols which stood for three combined concepts such as one+jar+oil. Such a symbol could be analyzed into its component parts because ease of incision allowed symbols for them to be generated.

When the symbol for ‘one’ became distinct it became the first purely numerical symbol. The same step created the conceptual distinction between, say, one jar, and jar, the latter concept being a greater abstraction than the former.

These distinct abstractions could be recombined into a more reduced system of notation.

The separation and recombination of symbols of different kinds led to a systematic method of reduction as follows. Suppose a merchant sells 10 spices A, B, C,…,J, in 3 grades, low, medium and high. Without analyzing an item into its type and grade 30 unrelated symbols would be needed to represent all possible items of merchandise. With analysis, we create three symbols, one for each grade, and ten symbols, one for each spice. Using them, 30 different symbols are reduced to combinations of 13 symbols (*e.g.* A1,A2,A3, B1,…,J3).

This modest reduction of magnitude 30/13 grows quickly if one supposes up to 1000 of 100 different commodities were traded. The comparison would then be between 100+1000=1100 words and 100×1000=100,000 word combinations. Instead of scribes being required to invent and remember 100,000 separate symbols, they need do so for 1100, which is about a hundred times less effort. The data reduction is 100,000/1100, or about 100.

THE EMERGENCE OF NUMBER SYSTEMS

To conceive of an object of thought it helps to be able to designate by word and symbol.

The emergence of distinct words and symbols for quantity represented the first and most basic stage in the full emergence of number as a concept. They could not fully exist in the mind before being able to designated as distinct objects of thought. As such, their properties could begin to be recognized and organized.

The next stage required the development of a notational system for numbers. Until that existed, any and all numbers could not be designated, and if not, could not be actual objects of thought.

Our modern system of notation requires a zero, and this took a remarkably long time to develop.

Our modern notational system for integers appeared in India during the 6th century CE, a few thousand years after the emergence of the first numerical symbols. Modern notation is not complicated, it is easily understood by any child: why did its development take so long? Why was it missed by the ancient world--by minds and civilizations that were able to develop sophisticated mathematics?

The central problem appears to have concerned zero. Modern notation treats zero and the other ciphers, 1 through 9, equally, whereas for the longest time the former was not viewed as a number at all. This view created a 'box' out of which it was very hard to see--so hard, in fact, that even after the adoption of this notation, zero was given only grudging acceptance as a number. Thus, in an early book on arithmetic its use in subtraction is described as follows:

When (in subtraction) nothing is left over, then write the little circle, so that the place does not remain empty. The little circle has to occupy the position, because otherwise there would be fewer places, so that the second might be mistaken for the first.[6]

The author here does not say that when nothing is left over in, say, the third decimal place, write a little circle to signify the *number* 0x100. That little circle, 0, is not yet a number—it is merely a placeholder, a reminder. And even as late as the 15th century zero was described as a symbol that merely causes trouble and lack of clarity.

The problem with zero that troubled thinkers was that they thought of number as describing a multiplicity of units of things whereas zero described, literally, no-thing. In fact, they were even troubled by the idea that one was a number, since that too does not describe a multiplicity. Some quotes[7] to this effect, from early Renaissance writers are as follows:

Unity is the beginning of all number and measure, for as we measure things by number, we measure number by unity.

Unity is not a number, but the source of number.

The shift to the modern view of zero, and to number in general, can be traced back to what is called the 'Algorithmic Revolution'.

The growth of commerce required the development of algorithmic arithmetic, a system of mechanical symbolic manipulation that required, in turn, the uniform perfection of the notational system for number.

Long division as learned in grade school illustrates what is meant by an algorithm. It is a set of rules for performing a calculation mechanically, without real thought, by rote: a procedure that can be done by a machine, or by a relatively few lines burned on a computer chip. Algorithm is a word derived from the name of the Arab mathematician Al-Kwarizmi who wrote the first book describing how to calculation mechanically using the Indian notational system. These methods became known collectively as algorithmic arithmetic--history's first example of methods classed as *symbolic manipulation*. Competing with it were older sets of methods, most prominently the set developed for computing on an abacus--a system of beads strung on wires--token manipulation. Practitioners were known as *algorithmi* and *abaci* respectively.

As each abacus bead is a token of unity, the changeover from abaci to algorithmi, is analogous to that between users of clay tokens and of inscriptions on clay tablets. Both led to unforeseeably large changes in human culture. The immediate reasons for the changeovers were also analogous: a growth of international commerce increased the demand for people with arithmetic skills, and algorithmic methods allowed ordinary people to do calculations that had previously been beyond them.

The simplicity of algorithmic methods is based on the uniform perfection of the notational system. In comparison, for example, the imperfect Roman system makes long division difficult. Algorithmic long division can be done semi-consciously, whereas in Roman notation it requires constant careful alertness.

Alfred N. Whitehead commented on the significance of this as follows

> *By relieving the brain of all unnecessary work, a good notation sets it free to concentrate on more advanced problems.... Before the introduction of the Arabic notation, multiplication was difficult, and the division even of integers called into play the highest mathematical*

faculties. Probably nothing in the modern world would have more astonished a Greek mathematician than to learn that, under the influence of compulsory education, the whole population of Western Europe, from the highest to the lowest, could perform the operation of division for the largest numbers...

By the aid of symbolism, we can make transitions in reasoning almost mechanically by the eye, which otherwise would call into play the higher faculties of the brain. It is a profoundly erroneous truism, repeated by all copy-books and by eminent people when they are making speeches, that we should cultivate the habit of thinking what we are doing. The precise opposite is the case. Civilization advances by extending the number of important operations which we perform without thinking about them.

By the mid 13^{th} century algorithmic arithmetic was established in Italy well in advance of anywhere else in Europe, the immediate reason for this being the commercial pre-eminence of the northern Italian cities. Dissemination was slow: although by the 15^{th} century one could learn algorithmic addition and subtraction in Germany, instruction in the arts of multiplication and division could still only be found in Italy.[9] By the beginning of the 16^{th} century, just before the scientific revolution, the Germans were still 200 years behind!

The algorithmic revolution, driven by the demands of commerce, expanded the meaning of number so as to include zero.

Before the advent of algorithmic arithmetic, a calculation required continual thought: remembering numbers, rules, and relations, adding, multiplying, and so on. It was a mental stream of concepts and operations in which occasionally a number might be written, but only as an aid to memory. The written symbol sat beside the stream.

In the switch to algorithmic arithmetic, calculation became symbolic manipulation, and a stream of symbols replaced a stream of thought. In the symbolic stream, 0 and 1, were very similar to the other (multiplicity denoting) ciphers. The algorithmi began to see the ciphers 0,…,9 as merely the atoms out of which the symbols for all integers were formed, and all integers began to be viewed in terms of their roles in symbolic manipulation.

The practice of the algorithmi effectively established a dual meaning for number. Today, all types of numbers (*e.g.* complex numbers) are computational symbols, while only some also are quantifiers (the natural numbers quantify discreta, the real numbers, continua). We now switch between the two meanings without much thought, and this has caused confusion over meaning. In 250 CE, the Greek mathematician Diaphantos, had already stated general rules for manipulating negative numbers: *Minus multiplied by minus gives plus, minus times plus gives minus…*, yet in 1545, the eminent mathematician Cardan still called negative numbers ‘absurd’. Similarly, today, physicists often represent time in a computation as an imaginary number, and some will say "time is imaginary". Without specifying meaning, this also appears absurd.

Number systems are infinite systems of symbols which transform autonomously; they operate upon and thereby transform into one another.

Algorithmic arithmetic depends on the ability of infinite sets of numbers to transform into one another: 1 transforms itself into 2 through the operation of addition, 7 transforms 9 into 63 through multiplication, and so on. A set of numbers plus a set of operations transforming them into members of the *same* set (allowed operations) defines a number *system*.

For example, the natural number system (1,2,...) allows only subtraction of lesser from greater. If you break this rule, the result is not part of the system; within its context, the result is meaningless. As a result, if you create an algorithm that uses the naturals, but requires subtractions, it is hard to ensure that an undefined symbol (1-2) will not appear during the calculation. The integer number system (…,-2, -1, 0, 1, 2,…) does not have this problem; it permits any subtraction.

Similarly, while multiplication is allowed for all pairs of natural numbers, division is not. As a result, if you create an algorithm that uses the naturals, but requires divisions, it is hard to ensure that an undefined symbol (such as 1/2) will not appear during the calculation. The rational number system does not have this problem.

Similarly, you are allowed to take the square root of 5 if you are working in the real number system but not in the rationals because √5 is not rational (it was called an '(ab)surd' number). And again, you are allowed the square root of -5 in the complex number system, but not in the real because √-5 is not real (it is called 'imaginary').

A number system's uniform simplicity of operations is the key its utility.

The number systems (the natural numbers, integers, rationals, reals, complex, quaternions) were created as people expanded the computational use of number. Like algorithms, they were created to eliminate unnecessary brain-work. Each system defines its allowed operations, and guarantees that they work together smoothly, like gears in a machine; they never lead to mathematical inconsistencies. No matter how long, a calculation is guaranteed to be correct as long one respects only its few simple prohibitions (*e.g.* in the complex number system you are allowed to take any complex root of any complex number except zero).

Allowed operations are also uniform: *e.g.* the rule for adding fractions (*i.e.* rational numbers) is the same for all. Lacking this property, the important algorithms of arithmetic (for adding long numbers, long division, for extracting a root), all built out of simple cycles, indefinitely repeated, could not be guaranteed. Calculations that had been long and often tricky were reduced to being merely long: a few simple steps mechanically repeated. This achievement of uniformity was made possible by the prior abstraction: the characteristics of number were reduced to only those relevant to their computational use. Striped away were the irrelevant properties associated with their origin as symbols of multiplicity.

Systems that lack the foregoing properties are complex. They are characteristically composed of many dissimilar parts that operate on each other in many dissimilar ways; in contrast, the infinitely many integers are all similar--each, just "one more of the same"--and they operate on each other in just a few, simple ways. A paradigmatic example of a complex system is a large scale computer application; typically, it can never be guaranteed to be bug free, whereas computational applications of number systems can be. One can say that a complex system is never completely understood, whereas an arithmetic algorithm is, always. As suggested, reasons for such differences reduce to the concept of reduction.

The ability of numbers to transform autonomously is key to their role in the physical sciences.

A properly running machine has all it parts moving exactly according to plan, without the necessity of an outside hand reaching in to fix or adjust--without a *deus ex machina*. A God or prime mover may be needed to supply power (and to design and build the machine), but otherwise its motion self regulating. Each gear and lever regulates the motion of the other. Such a machine can be said to run *autonomously.*

The atoms of numerical calculation are the ciphers 0,1,…,9. The molecules that determine material properties are the numbers formed from the ciphers, be they the integer, real, or complex. The gears and levers are the basic operations defining transformations amongst the numbers. The machine design is the algorithm. The source of power (but not intelligence) is the human computer (or the electric power running the electronic computer).

Once numbers to be input to an algorithm are set, and the power is turned on, then, without thought or intervention, a stream of symbols appears. Just as gears and levers change each other, so do the numbers produced during the calculation. The numbers transform autonomously according to the sequence of operations allowed by system and prescribed by algorithm.

As we shall see, this kind of symbolic autonomy is essentially unique to number, and is key to its role in the physical sciences.

NUMBER SYSTEMS AND PHYSICAL SYSTEMS

The natural numbers may be defined abstractly and uniquely.

The history of number is one of successive abstractions; the last of them, formulated in the late 19th century, abstracted structure from notation. It defined the set of natural numbers in terms of two primitive (*i.e.* not further definable) words/ideas: (i) that of a *first* and (ii), that of a *successor*. The set of natural numbers has these defining properties: (i) it has a first element, (ii) every element except the first is the successor of an element, (iii) the first is not the successor to any element, (iv) no element has more than one successor, and (v) none can be the successor of more than one element.

We can imagine many sets of discrete elements with these properties: the set of all instants in time starting from the first, the set of all humans as ordered by birth starting with Adam, the set of all theorems of arithmetic alphabetically ordered, and so on. No notational system is needed to imagine their structure.

But we could number them if we wished (first, second,…). They are all *numerable.* This suffices to show that their structures are the same.

We can superimpose additional structure on these sets. In fact, it is natural to think of humans and theorems in terms of multiple ancestors--parents in one case, predicates in the other. In principle, a notation reflecting ancestry could be constructed out of the natural numbers. In fact, all types of number can be constructed from the naturals, even non-denumerable numbers such as the reals. Thus, one can say that the naturals form the *simplest* and most *basic* number set. In this sense, they are *unique*.

Why Pythagoras was right, and number is the language of science.

The foregoing, when supplemented by another important idea, can provide an understanding of why Pythagoras was correct in stating that *all is number.* The supplementary idea--remarkably, found already throughout early Greek natural philosophy--states that physics cannot tell us *what is* but only how things *change.* Here, change refers to both properties and place, *i.e.* it includes motion. 'What is', in this context, refers to composition but not conformation. It asks for components, but not how they conform themselves. Conformation falls under the heading of change: how components arrange themselves in positions and orbits.

Thus, if one asks, what *is* water? The answer is Hydrogen and Oxygen. Then, what *is* Hydrogen? A proton and an electron?…then what *is* an electron? An elementary particle?…then what *is* that? That just *is*; 'elementary' means we cannot say of what it is made.

If you claim that the electron is not elementary and that you know of what it is made (*e.g.* strings, energy,…?) you just get to the same question down a level. Eventually, there is an end to it; a final name. It could be some other elementary particle (or basic matter, field, or energy)--any kind of basic stuff. Whatever it is called, it just labels a set of associated properties, and these just specify interactions. (For example, an electron's properties of mass and charge help specify its interactions with gravitational and electromagnetic fields.) But as the sole purpose of interactions is to determine change, asking what something *is,* leads only to information about how it changes; it cannot lead to qualitatively different information.

In sum, the endpoint of a physical inquiry into what something *is,* can be no more than a catalog of its constituent elementary parts--a catalog of names whose meaning consists solely in information about how things change.

The word 'interaction' combines internal and action; when things interact, the action is internal, and internally determined. There is no external determinate (external to the set of interacting things) of change. Interacting things therefore change *autonomously*: according to their own (natural) law. This (standard) use of the word interaction by Physics reflects its sole concern with *natural* change, nature being definable as just that part of existence that changes autonomously, whereas supernatural means beyond nature and its laws. Autonomy is a powerful idea. It instructs us that natural change is regulated without reference to anything external to nature, and this includes, in particular, without reference to gods, or to God.

If, then, the basic stuff of nature changes autonomously, then the scientific symbols representing that stuff must do so also. The central role of autonomous change in physics, and the essential uniqueness of symbolic autonomy to number, together, form part of the reason that *All is Number*. Further parts will emerge as we proceed.

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