Chapter 7

The Optics of Thin Lenses

7.1 Purpose

In this experiment, the formation of images by convex lenses will be explored. The application of the thin lens equation and the magnification equation to single lenses systems will be investigated. In a later lab (The Telescope), these properties will be used to build and measure a small telescope.

7.2 Introduction

7.2.1 The Thin-Lens (Gauss’) Equation

Light can travel through many transparent media such as water, glass and air. When this happens the speed of light is no longer \( c = 3 \times 10^8 \text{ m/s} \), (the speed in vacuum), but is less. This reduction in the speed causes the light to change directions. This change in speed and direction is called refraction. The number which is used to describe how much the light’s speed and direction changes is called the ‘index of refraction’ and depends on the material. Air has an index of refraction which is almost 1.0003 while glass has an index of refraction of approximately 1.5. Diamond has an index of refraction of 2.4 which accounts for the brilliant sparkles of diamond jewelry. Refraction is used in many items of modern life from eye glasses to fiber optic communication cables.

Lenses are common optical devices constructed of transparent material e.g. glass or plastic, which refract light in such a way that an image of the source of light is formed. Normally, one or both sides of the lens has a spherical curvature. When parallel light from a source impinges on a converging lens, the parallel rays are refracted so that all the light comes together at a focal point. The distance between the lens and the focal point is called the focal length of the lens. An imaginary line parallel to the light rays and through the center
of the lens is called the **principal axis**. A lens is considered ‘thin’ when the focal length is significantly larger than the thickness of the lens.

Another basic type of lens is the diverging lens or concave lens. With a diverging lens, parallel rays are spread out by the lens. The focus of a diverging lens is on the same side of the lens as the impinging parallel rays. We will not need concave (diverging) lenses for our study of optics.

The **thin-lens equation** relates the distance of the object from the lens, $d_o$, and the distance of the image from the lens, $d_i$, to the focal length of the lens, $f$.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

The **magnification equation** is:

$$m = \frac{\text{Image Height}}{\text{Object Height}} = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

where $h_o$ is the object height, $h_i$ is the image height. The magnification, $m$, is the ratio of these heights. Since the triangle formed by the ray through the center of the lens and the object distance and height is a similar triangle to the triangle formed by the ray through the center of the lens and the image distance and height, the ratio of $\frac{h_i}{h_o} = -\frac{d_i}{d_o}$.

The following sign conventions are used with the thin-lens and magnification equations:

- $f$ is positive (+) for a converging lens. ($f$ is negative (-) for a diverging lens.)
- $d_o$ is positive (+) when the object is to the left of the lens (real object). $d_o$ is negative (-) for an object to the right of the lens (virtual object).
- $d_i$ is positive (+) for an image formed to the right of the lens for a real object. $d_i$ is negative (-) for an image formed to the left of the lens for a real object.
m is positive (+) for an image that is upright with respect to the object. m is negative (-) for an image that is inverted with respect to the object.

When more than one lens is used, the thin lens equation can be applied to find the image location for the first lens. This location of the image from the first lens is then used as the object for the second lens and a second application of the thin lens equation.

### 7.3 Procedure

**Special Cautions:**

- Do not drop the components or touch the optic surfaces.
- Be careful when moving around the darkened room.

#### 7.3.1 Simple Focal Length in One Dimension

If parallel rays of light impinge on a semi-circular piece of plastic, the rays will refract and come to a focus at some point. To illustrate how a lens works in one dimension, we will use a semicircular (half cylindrical) disk of plastic. The setup for this part of the experiment is shown in figure 7.1

![Figure 7.1: Arrangement of the ray table and holder on the optics rail. The parallel ray lens and slit plate are attached to the holder between the light source and ray table. The cylindrical lens is on the ray table with the round side towards the light source.](image)

- The components are magnetic and will stick to the black component holders. Attach the parallel ray lens to one side of a component holder. Attach the slit plate to the other side of the component holder.
- Place the light source at one end of the rail. Place the component holder with the parallel ray lens and slit plate approximately 10 cm in front of the light source. Place the ray table and base approximately 15 cm in front of the component holder. The ray table base should have the smaller (lower) end towards the light source.
- Place the cylindrical 'lens' on the ray table and align the cylindrical lens so the round side is towards the light source and centered on the ray table. (The marks on the ray
table should be used to align the flat side of the cylindrical lens so it is perpendicular to the long axis of the rail.)

- Parallel light rays should impinge on the circular side of the cylindrical 'lens'. Using a small white card, look for the 'focus' of the lens. As you move the card back and forth, you should see the rays converge into a single line. (Ignore the rays above the cylindrical lens which do not change the spacing.) Using a ruler, measure the 'focal length' (distance from the 'lens' to the focal point) of this one dimensional lens. Record the value below.

  Approximate focal length of cylindrical 'lens' ____________

- If you reverse the cylindrical lens so the flat side is facing the light source, can you still find a focus? Explain below why or why not the lens in the configuration should have a focus.

- Remove the ray table and base, parallel ray lens and slit plate from the rail to a safe place.

Figure 7.2 shows the equipment used for the rest of the experiment. The cross arrow target can be mounted on the incandescence light source. The equipment includes a viewing screen with a scale and lenses of various focal lengths. The lenses and other components have magnetic strips on the back so they attach easily to the component holders. Note the distance scale along the edge of the magnet rail. The position of the lens on the holder can be determined by the notches on the component holder (See figure 7.3)

Figure 7.2: The left photo shows the lenses, viewing screen, cross arrow target and a component holder. The right photo shows the light source, lenses and viewing screens on the rail.
The position of the lens can be determined by reading the scale on the side of the rail. The position is different depending on how the lens is attached to the component holder.

### 7.3.2 Procedure for Approximate Focal Length of a Convex Lens

The thin lens equation relates the object and image distances to the focal length of the lens. If the object distance can be made very large \( \left( \frac{1}{d_o} = \frac{1}{\infty} = 0 \right) \) then the equation reduces to the image, which will be very small, being at the focal length of the lens.

* Place the white viewing screen at the far end of the magnetic optical rail. Place the cross arrow on the light source at the other end of the rail.
* Place the 75 mm (0.075m) convex lens on a magnetic holder at the far end of the magnetic optical rail near the screen (see figure 7.4). The lens should be centered on the opening in the holder.
* Adjust the lens until a sharp very small image is formed. Since the object i.e. the cross arrow target is far away, the distance between the lens and the screen is approximately the focal length of the lens. Record the focal length of the lens and object distance.

**Focal length of lens**

**Object distance**

* Explain why this approximate focal length method works using the thin lens equations.

### 7.3.3 Procedure for the Focal Length of a Single Convex Lens

The magnetic optics rail contains a measuring scale along the side. The optics components mount on magnetic component holders. Each holder has a small indicator position on the
Figure 7.4: Arrangement of components for the approximate focal length measurement for a single lens

side to locate the position of the component on the rail side scale (see figure 7.3. Each component should be centered on the holder.

- Attach the cross arrow target (CAT) to the front of the light source. Place the the light source at one end of the rail. Place the 75 mm (0.075m) focal length lens on a holder and position it 30 cm (.3 m) from the CAT. See Figure 7.5
- Calculate the location of the image formed by the lens using the thin lens equation. Calculate the size and orientation of the image using the magnification equation. Show your calculations below
- Place the view screen on a holder and place it at the calculated image position. Make small changes to the screen position to get a good focus.
- Record the measured object and image distance below.

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<thead>
<tr>
<th>Object distance</th>
<th>Image distance</th>
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- Measure and record below the image size and orientation. (The circle on the CAT is 1 cm in diameter.)

<table>
<thead>
<tr>
<th>Image size</th>
<th>Image orientation</th>
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- Do the calculated results agree with the measured results? Calculate the percentage error between the calculated and measured value of the focal length.

<table>
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<th>Percentage error</th>
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- In the thin lens equation, the sum of the inverses of the object distance and image distance are equal to the inverse of the focal length. If the image and object distance are interchanged, another image should be observed on the screen assuming the total
distance between object and image is not changed. Move the lens so the distance be-
tween the screen and the lens is .1 m (10cm). Is a sharp image formed by interchanging 
the object and image distance. What happens to the magnification? Explain

7.3.4 Questions

1. Explain how a lens uses refraction to bend light. Why is one or both of the surfaces 
of the lens 'spherical'

2. Explain what the terms focal length and focal point mean.

3. In the 'Procedure for Approximate Focal Length' the light rays were not at an infinite 
distance and thus not parallel to the principle axis. Use the thin lens equation to 
estimate how accurate you expect this type of measurement to be.

4. A beacon in a lighthouse is designed to project a beam of parallel light rays. The 
beacon consists of a small intense light bulb and a large converging lens. Should the 
light bulb be placed at a distance greater than the focal length of the lens, at the focal
point of the lens or a distance less than the focal length of the lens?

7.4 Conclusion

Write a conclusion about what you have learned. Include all relevant numbers you have measure with errors. Sources of error should also be included.