Chapter 3

Oscillations: Mass on a Spring and Pendulums

3.1 Purpose

3.2 Introduction

Galileo is said to have been sitting in church watching the large chandelier swinging to and fro when he decided that the period of that 'pendulum' was constant, independent of the amplitude of oscillation. After some thought, you might ask if Galileo could have been wearing a watch with which to measure the period. In fact, we are told, Galileo used his own pulse as a clock to time the swinging chandelier.

A periodic motion is one that repeats itself in a fairly regular way. Examples of periodic motion are a pendulum, a mass attached to a spring or even the orbit of the moon around the earth. The time it takes for one full motion, or cycle, is known as the **period**. For example, to complete one full cycle the moon requires approximately twenty-eight days. Thus the period of the moon is twenty-eight days. The period is normally represented by the letter T. A quantity related to the period is the frequency. The **frequency** is the number of oscillations per second. The frequency, f, and period are related by:

$$f = \frac{1}{T} \tag{3.1}$$

Another quantity which describes the periodic motion is the **amplitude**. The amplitude is the maximum displacement from the equilibrium position of the motion.

A particular type of periodic motion where the motion repeats in a sinusoidal way is called simple harmonic motion. Two examples of simple harmonic motion are the oscillations of a pendulum when the displacement is small and the oscillation of a mass on a spring. In this lab, simple harmonic motion will be examined using a mass on a spring and using pendulums.

When a spring is stretched or compressed, the force required to stretch the spring is given by Hooke's Law:

$$F = -kx \tag{3.2}$$

where F is the force, x is the distance the spring is stretched or compressed and k is the spring constant. The spring constant determines the 'stiffness' of the spring and is measured in Newton/meter. The minus sign indicates the force points in the direction opposite to the displacement.

The normal method of analyzing the motion of a mass on a spring using Newton's 2^{nd} leads to a differential equation which is beyond the scope of this course. However, we can state the result for the period of a mass on a spring as:

$$T = 2\pi\sqrt{\frac{m}{k}} \tag{3.3}$$

where k is the spring constant for the spring and m is the oscillating mass.

A simple pendulum is a mass on a string or wire. The mass is assumed to be small in size. For a real pendulum, we use the center of mass of the mass to account for the fact the mass is not a point. We also assume the string or wire does not stretch. The period for a simple pendulum is given by:

$$T = 2\pi\sqrt{\frac{L}{g}} \tag{3.4}$$

where L is the length of the pendulum and g is the acceleration of gravity (9.81 m/s^2). It may be somewhat surprising that the mass does not enter into the equation for the period. We will verify this during this lab. It should also be noted that the derivation of the formula for the period of a pendulum assumes the amplitude of the motion is small (less than 5^o from the vertical).

3.3 Equipment:

String, spring, masses, mass hanger, photo-gate timer, meter stick and protractor.

3.4 Procedure

3.4.1 Mass on a Spring

We will first determine the spring constant of the spring but placing different forces on the spring and measuring the stretching of the spring. Then we will observer the period of oscillations for a few different masses. The basic apparatus is shown in figure 3.1

Figure 3.1: Basic apparatus for the mass on a string.

- **Determining the Spring Constant:** Attach the mass hanger to the spring hanging from the arm of the stand. Make sure the hanger is not in motion. Measure the distance between the bottom of the hanger and the top of the table.
- Add a 0.1 kg mass (100 grams) to the weight hanger. Measure the distance the spring stretches with the additional 0.1 kg of mass. Record the mass and the amount the spring stretches in a table like table 3.1.
- Continue adding 0.1 kg masses up to 0.5 kg. For each addition of mass, measure the stretch of the spring and record the values. **Do not exceed 0.5 kg (500 grams) or stretch the spring out of shape**.
- Multiple each mass in kilograms by g (9.81 m/s^2) and record the value in the table. Plot force (y axis) verse stretch in meters (x axis). Draw a best fit line to the data points and calculate the slope of the line. The slope of the line is the spring constant k. Why?
- **Measuring the Period of Oscillation:** Place masses on the weight hanger so the hanger plus the masses have a total mass of 0.2 kg (200 grams). Attach the hanger to the spring hanging from the arm of the stand.
- Gently stretch the spring approximately 5 centimeters (0.05 m) by pulling down on the weigh hanger and masses. Release the masses so the mass oscillates vertically.
- Using the seconds hand on the clock or your watch, measure the time for 15-20 oscillations of the mass. Divide the total time by the number of oscillations to determine the period.
- Calculate the percentage difference between the measure period and the period calculate from equation 3.3.

mass (kg)	force (Newton)	stretch (m)

Table 3.1: Example Table for determining the spring constant

Figure 3.2: Basic apparatus for the pendulum part of the experiment.

3.4.2 Pendulum

In this section we will measure the period of different masses and different pendulum lengths. An electronic photo-gate timer will be used to measure the period of the different pendulums. The basic apparatus is shown in figure 3.2

- Remove the spring and mass hanger from the arm of the stand. Attach a string of approximately 0.75 meters to the arm of the stand. Attach a 0.05 kg (50 gram) mass to the end of the string.
- Turn on the photo-gate timer and set the switch to pendulum ('pend'). Position the photo-gate so the mass swings through the photo-gate without hitting the photo-gate. Hold the pendulum to one side and push the 'reset' button on the timer. Release the pendulum from a small angle (5^o or less from the vertical). The timer will stop after one complete period. Record the time. Repeat the measurement 5 times and find the average period. Calculate the period from equation 3.4. Calculate the percentage difference between the average measured value and equation pend:period.
- Replace the 0.05 kg mass with a 0.1 kg (100 grams) mass. Measure the period 5 time and average the values. Replace the 0.1 kg mass with a 0.2 kg (200 gram) mass and repeat the measurement. Does the period change with the larger masses?

- Replace the string with a length of approximately 0.5 meters. Use the 0.1 kg mass. Adjust the height of the arm on the stand to position the pendulum to swing through the photo-gate. Measure the period of this pendulum 5 times and average the values. Calculate the period from equation 3.4. Calculate the percentage difference between the measured values and equation 3.4.
- Repeat the measurement and calculation with a pendulum of approximately 0.25 meter length.

3.5 Questions

- With the mass on the spring, why do we measure 10-15 periods and not one?
- Should the maker of a pendulum clock be concerned with the expansion of the material of the pendulum as the temperature changes?
- Would the period of a 1 meter pendulum on the moon (where the acceleration of gravity is smaller) be larger, smaller or the same. Would the period of a mass on a spring be larger, smaller or the same on the moon, assuming the spring and mass are the same.
- The formula for the period of a mass oscillating on a spring is independent of the amplitude (like with the pendulum). When the mass undergoes larger oscillations, how can the period stay the same since the mass must travel further?