

# Chapter 1

## Vectors, Forces, and Equilibrium

### 1.1 Purpose

The purpose of this experiment is to give you a qualitative and quantitative feel for vectors and forces in equilibrium.

### 1.2 Introduction

An object that is not accelerating falls into one of three categories:

- The object is static and is subjected to a number of different forces which cancel each other out.
- The object is static and is not being subjected to any forces. (This is unlikely since all objects are subject to the force of gravity of other objects.)
- The object is moving with **constant velocity**. In this case, the object may be subject to a number of forces which cancel out or no force at all. This case is not considered in this lab.

The category of physics problems that involve forces in static equilibrium is called statics. Physicists and engineers are subjected to static problems quite frequently. A few examples of these principles in use are seen in the design of bridges and the terminal velocity of a person falling through the air.

Mathematically, forces in equilibrium are just a special case of Newton's Second Law of Motion, which states that the sum of all forces is equal to the mass of the object multiplied by the acceleration of the object.

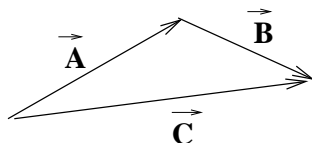
The special case of forces in equilibrium (static), occurs when the acceleration of the object is zero. When this situation arises, Newton's Law becomes:

$$\Sigma \vec{F} = 0 \tag{1.1}$$

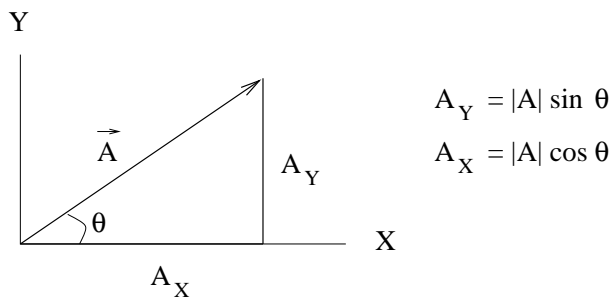
This equation simply states that the sum of all of the force vectors acting on an object is equal to zero when the object is in equilibrium (static or not accelerating). We will use an apparatus called a 'force table' to demonstrate static forces and equilibrium.

Note that equation 1.1 is a 'vector' equation. A vector is a quantity that has both magnitude and direction. A quantity which does not have direction is called a scalar quantity. In order to distinguish vectors from scalars, vectors are identified with either bold face or an arrow on the top of the letter:  $\mathbf{V}$  or  $\vec{V}$ . A vector can be represented by an arrow with length proportional to the magnitude. The direction of an arrow gives the direction of a vector.

There are two basic ways to add vectors: graphically and by components. To add vectors graphically, you place the tail of one on the head of the other. A vector from the tail of the first to the head of the second vector is the resultant (sum). For two vectors  $\vec{A}$  and  $\vec{B}$ , the resultant  $\vec{C} = \vec{A} + \vec{B}$  is shown below.



The second method of adding vectors is to add their components to get the components of the resultant vector.



Once you find all the component you add the x components to get the resultant x component and likewise for the y components. The magnitude of the resultant can be found from the Pythagorean theorem:

$$R = \sqrt{R_x^2 + R_y^2} \quad (1.2)$$

where  $R_x$  is the x component of the resultant and  $R_y$  is the y component of the resultant. The direction of the resultant (sum) can be found from:

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) \quad (1.3)$$

Note that calculator arctan ( $\tan^{-1}$ ) functions may not determine the correct quadrant of the vector. This is easily determined by the signs of the x and y components in the ratio.

### 1.3 Equipment:

Force table, masses, weight holders, pulleys, string.



Figure 1.1: The photograph shows the force table with the the pulleys, weight hangers and slotted weights.

## 1.4 Procedure

The force table apparatus is shown in figure 1.1. A screw under the pulley is used to secure the pulley to the edge of the force table. Make sure the string goes over the wheel of the pulley.

- **Two Forces in equilibrium:** Place one pulley at zero degrees on the force table and add 100 grams (0.1 kg) to the mass hanger at the end of the string. The total mass will be approximately 150 grams (0.15 kg) since the mass hanger has a mass of 50 grams (0.05kg). Weigh the hanger and mass to determine the exact mass. The position of this pulley at  $0^\circ$  should not be moved for the rest of the experiment.
- With the center pin in place, position another pulley with 150 grams (0.15 kg) of mass on the string somewhere on the table. The total mass of the second mass hanger can be adjusted with small masses to be nearly equal to the mass of the first mass hanger ( $\approx 0.15$  kg). Alter the angle of this pulley until the center ring does not move when the center pin is pulled out. (See figure 1.2) The two forces, in the form of tension on the strings, cancel each other out when this occurs. Record the angular position of the second pulley.
- **Three forces in equilibrium:** Prepare a third mass hanger with a total mass of 0.15 kg. Place this mass over the third pulley. Keep the first mass at  $0^\circ$ , adjust the second and third masses until the system is in equilibrium.
- Using graph paper, a protractor and ruler, add the three force vectors graphically. The force is the total mass times the acceleration of gravity ( $g = 9.8 \text{ m/s}^2$ ) Is the sum of the three forces zero?

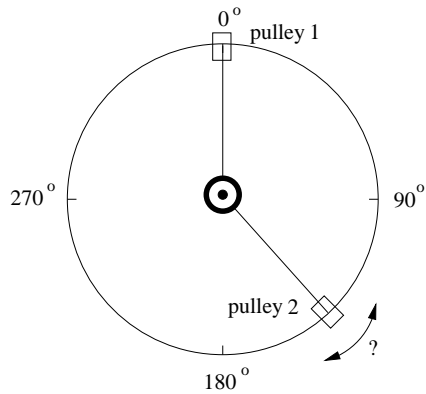


Figure 1.2: Adjust the angle of pulley 2 so the force from the weight over pulley 1 is in equilibrium with the force from the weight over pulley 2 .

- Use the component method, add the three force vectors. Is the sum of the three vectors zero using the component method?

## 1.5 Questions

1. If you have a total of two pulleys on the force table, one with a mass of ( $m$ ) hanging from it and the other having a mass of ( $2m$ ), where would the two pulleys have to be placed in order for the forces to cancel?
2. When an object is hanging from a string in equilibrium, the force pulling it down is gravity, and the force holding it up is called tension. For the object to be in equilibrium, the force due to gravity must be equal to the force of tension in the rope. What are the tensions on each of the ropes shown in the figure below? (Hint: Use the Pythagorean Theorem.)

