## Chapter 6

## Capacitors and RC Circuits

$\qquad$ Section:

### 6.1 Purpose

The purpose of this experiment is to investigate the physics of capacitors in circuits. The charging and discharging of a capacitor is the actual movement of electrons into and out of the capacitor. You will measure the voltage across the capacitor as it charges and discharges. You will study capacitors connected in series and in parallel.

### 6.2 Introduction

The voltage $(V)$ across a charged capacitor is proportional to the amount of charge $(Q)$ on the plates of the capacitor and inversely proportional to its capacitance $(1 / C)$. Capacitance is most easily thought of as the amount of charge a capacitor will hold for a given voltage.

$$
\begin{equation*}
Q_{0}=C V_{0} \tag{6.1}
\end{equation*}
$$

with $Q_{0}$ being the charge in Coulombs, $V_{0}$ the voltage measured across the capacitor, and $C$ the capacitance in Coulombs/volt. The unit Coulombs/volt is defined as a Farad (F).

Consider a capacitor with capacitance C charged with a charge of $Q_{0}$. If the capacitor is discharged through a resistor with a resistance of R ohms, conceptually the time for the charge to neutralize (move from one plate to the other) will depend on the value of the resistor. The charge will neutralize rapidly if the resistor has a small value and slower if the resistor is larger.

Now consider this situation quantitatively. Using Ohm's Law $(V=I R)$, we can write the charge on the capacitor in a closed circuit with a resistor, $R$, at any instant as:

$$
\begin{equation*}
Q=C I R \tag{6.2}
\end{equation*}
$$

Since

$$
\begin{equation*}
\frac{\Delta Q}{\Delta t}=I \quad\left(\frac{d Q}{d t}=I\right) \tag{6.3}
\end{equation*}
$$



Figure 6.1: Time dependence of charge in a RC circuit for charging and discharging.
the right hand side contains the rate of change (or derivative) of the charge with respect to time. Thus,

$$
\begin{equation*}
Q=C R \frac{\Delta Q}{\Delta t} \quad\left(Q=C R \frac{d Q}{d t}\right) \tag{6.4}
\end{equation*}
$$

This differential equation can be solved to give:

$$
\begin{equation*}
Q=Q_{0} e^{-\frac{t}{R C}} \tag{6.5}
\end{equation*}
$$

where $Q_{0}$ is the initial charge on the capacitor (at time $t=0$ ). The quantity $R C$ is called the time constant, $\tau$, of the resistor-capacitor circuit. When $t=R C$, the charge on the capacitor is $Q=Q_{0} / e$. In other words, when the capacitor is discharging at time intervals equal to the time constant of the circuit, the charge on the plates of the capacitor reduces by $e$.

Now consider the situation where a capacitor and resistor are connected to a voltage source of $V_{0}$ volts. Conceptually the capacitor will charge and the rate of charging will depend on the value of the capacitor and resistor. In this case, the charge of the capacitor is given by

$$
\begin{equation*}
Q=Q_{0}\left(1-e^{-\frac{t}{R C}}\right) \tag{6.6}
\end{equation*}
$$

In both the charging and discharging situation, the voltage as a function of time across the capacitor is directly proportional to the charge from the definition of capacitance. Figure 6.1 graphically shows the charge on the capacitor in an RC circuit for charging and discharging.

Now consider capacitors connected in series and in parallel (see Figure 6.2). For two capacitors in series ( $\mathrm{C}_{1}$ and $\mathrm{C}_{2}$ ), the total capacitance $C_{S}$ is given by:

$$
\begin{equation*}
\frac{1}{C_{S}}=\frac{1}{C_{1}}+\frac{1}{C_{2}} \tag{6.7}
\end{equation*}
$$

For two capacitors in parallel $\left(\mathrm{C}_{1}\right.$ and $\left.\mathrm{C}_{2}\right)$, the total capacitance $C_{P}$ is given by:

$$
\begin{equation*}
C_{P}=C_{1}+C_{2} . \tag{6.8}
\end{equation*}
$$



Figure 6.2: Series (left) and parallel (right) capacitors.


Figure 6.3: The pre-configured RC circuit box connected to the 550 universal interface 'signal generator' (DC voltage source).

### 6.3 Procedure

## Special Cautions:

- The capacitors in this experiment can be damaged if the voltage is set above 6 volts. Do not exceed 4 volts.

The circuits for this experiment are pre-configured in a box with with appropriate connectors (see Figure 6.3). A schematic of the box is shown in Figure 6.4.

### 6.3.1 RC Time Constant with a Single Capacitor

- Open the file "RC Circuits". A voltage vs time plot will appear. 'Click' on the signal generator icon (出) on the tool panel and a signal generator will appear. The signal generator in 'DC' mode will be used as the voltage source. Connect the negative side of the signal generator output (ground) to the black input on the side of the circuit box. Connect the positive side of the signal generator to the red connector on the side of the circuit box.
- Connect a banana lead from the 'A' connector to the 'B' connector.


Figure 6.4: Schematic diagram of the Capacitor Circuit Box

- Connect the red lead of the voltage probe to the capacitor box terminal V1 and the black lead to capacitor box terminal V2. (Note: The banana plugs can be 'stacked' on top of one another.)
- Click the 'Start' button. Click the 'On' button on the Signal Generator. Push and hold the toggle switch to the right. This will connect the voltage source to the circuit and charge the capacitor.
- Observe the voltage on the graph. The voltage should exponentially rise to about 4 volts and remain constant. When the voltage seems constant, stop data collection.
- Calculate $R C$ from the values of the the resistor $(100 \mathrm{~K} \Omega)$ and capacitor (56 $\mu \mathrm{F})$.

Product of R.C $\qquad$

- Select the region representing the charging of the capacitor. Fit the data with an 'Inverse Exponent' $(\mathrm{A}(1-\exp (-\mathrm{B} \cdot \mathrm{x}))+\mathrm{C})$ curve to fit the data. The coefficient of the exponent (B) is equal to $\frac{1}{R C}$. Calculate the percent difference between the two values for the time constant for the charging circuit. Save the graph with the fit for your lab report

Fit parameter 'B' $\qquad$
$\tau$ from fit parameter $\qquad$
\% difference between fit $\tau$ and product RC

- Charge the circuit by holding the switch to the right for $\approx 20$ seconds. Start collecting data and push the switch all the way to the left. Stop when the voltage returns to zero and is nearly flat.
- Fit the plot using a 'Natural Exponential Fit' $(A \exp (B \cdot x)+C)$. The coefficient of the exponent $(\mathrm{B})$ is equal to $\frac{1}{R C}$. Calculate the percentage difference between the fit and the calculated value of RC .

Fit parameter 'B' $\qquad$
$\tau$ from fit parameter $\qquad$
\% difference between fit $\tau$ and product RC $\qquad$

- Save the data as a '.cap' file and send the appropriately labeled file to your TA.


### 6.3.2 RC Time Constant with Two Capacitors in Parallel

- Connect another banana lead between the 'B' connector and the ' C ' connector. This sets up the parallel capacitor circuit. The voltage probe leads should remain attached to the 'V1' and 'V2' positions.
- Calculate the total capacitance of the two parallel capacitors and the time constant, $\tau=R C$. Compare the fit values for both charging and discharging time constants to the calculated values.

Total capacitance of parallel capacitors $\qquad$
Time constant $(\tau)$ for capacitors in parallel $\qquad$

- Repeat the procedure from the single capacitor. Fit the charging circuit with the 'Inverse Exponent' fit and the discharging circuit with the 'Natural Exponential Fit'.


## Charging:

$\tau$ from fit parameter
\% difference between fit $\tau$ and product RC

## Discharging:

$\tau$ from fit parameter $\qquad$
\% difference between fit $\tau$ and product RC

### 6.3.3 RC Time Constant with Two Capacitors in Series

- Remove the banana wire between 'B' and 'C'. Connect between 'V2 and 'V3' with a banana wire. Move the banana wire connecting 'A' and 'B' to connect between 'A' and 'D'. This connects two capacitors in series. Finally, connect the voltage probe across 'V1' and 'V4' for this measurement.
- Calculate the total capacitance of the two series capacitors and the time constant, $R C$.

Total capacitance of series capacitors $\qquad$
Time constant $(\tau)$ for capacitors in series

- Repeat the procedure from the single capacitor fitting the charging circuit with the 'Inverse Exponent' fit and the discharging circuit with the 'Natural Exponential Fit'.


## Charging:

$\tau$ from fit parameter $\qquad$
$\%$ difference between fit $\tau$ and product RC

## Discharging:

$$
\begin{gathered}
\tau \text { from fit parameter } \\
\% \text { difference between fit } \tau \text { and product RC }- \\
\hline
\end{gathered}
$$

### 6.3.4 Questions

1. Using dimensional (unit) analysis derive the units of $R C$.
2. Using Kirchhoff's rules and the definition of capacitance, derive the formulas for total capacitance for series and parallel connections.

### 6.4 Conclusion

Write a detailed conclusion about what you have learned. Include all relevant numbers you have measured with errors. Sources of error should also be included.

