Chapter 9

Simple Harmonic Motion

Name:_____   **Lab Partner:**_____   **Section:**_____

9.1   Purpose

Simple harmonic motion will be examined in this experiment.

9.2   Introduction

A periodic motion is one that repeats itself in a fairly regular way. Examples of periodic motion are a pendulum, a mass attached to a spring or even the orbit of the moon around the earth. The time it takes for one full motion, or cycle, is known as the period. For example, to complete one full cycle the moon requires approximately twenty-eight days. Thus the period of the moon is twenty-eight days. The period is normally represented by the letter T.

A particular type of periodic motion where the motion repeats in a sinusoidal way is called simple harmonic motion. Two examples of simple harmonic motion are the oscillations of a pendulum when the displacement is small and the oscillation of a mass on a spring. In this lab, simple harmonic motion will be examined using a mass on a spring.

When a spring is stretched, the force required to stretch the spring is given by Hooke's Law:

$$F = -kx \tag{9.1}$$

where F is the force, x is the distance the spring is stretched or compressed, and k is the spring constant measured in N/m. The minus sign indicates the force points in the direction opposite to the displacement.

Dimensional analysis can be used to determine the proportionality of the period to the mass and spring constant. The result is $T \propto \sqrt{\frac{m}{k}}$.

To analyze the motion of a mass suspended on a spring, we start with Newton's 2nd law and Hooke's law:

$$F = ma = -kx \tag{9.2}$$

The acceleration is not constant but depends on the position so the normal kinematics equations for constant acceleration do not apply. In fact, this equation is a second order differential equation:

$$-kx = m\frac{d^2x}{dt^2} \tag{9.3}$$

Solving differential equations is beyond the scope of this course. However, we can state the solution and examine the properties of the solution. The solution (function) which satisfies this differential equation is:

$$x(t) = Asin(\sqrt{\frac{k}{m}}t + \phi) \tag{9.4}$$

where A is the amplitude (maximum displacement), ϕ gives the 'phase' of the motion at its beginning (t=0) and $\sqrt{\frac{k}{m}}$ is the angular frequency, ω. The angular frequency (in radians/s) is related to the frequency and period by:

$$\omega = 2\pi f = \frac{2\pi}{T} = \sqrt{\frac{k}{m}} \tag{9.5}$$

The velocity and acceleration can be determined from the position equation using calculus as:

$$v(t) = A\omega cos(\omega t + \phi) \tag{9.6}$$

and

$$a(t) = -A\omega^2 sin(\omega t + \phi) \tag{9.7}$$

Using the equations for position and velocity, the energy of the system can be analyzed. The potential energy of the spring is $\frac{1}{2}kx^2$. The kinetic energy of the mass is $\frac{1}{2}mv^2$. The total energy is then:

$$E_{total} = KE + PE = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \tag{9.8}$$

Figure 9.1: This photograph shows the harmonic motion setup. The force sensor and spring are at the top. The weight hanger should be placed directly over the motion sensor.

Substituting equation 9.6 for the velocity, v, and equation 9.4 for the position, x, results in:

$$E_{total} = \frac{1}{2}kA^2 sin^2(\omega t + \phi) + \frac{1}{2}mA^2\omega^2 cos^2(\omega t + \phi) \tag{9.9}$$

Since k = $m\omega^2$ from equation 9.5 and $cos^2\theta + sin^2\theta = 1$ we have:

$$E_{total} = \frac{1}{2}kA^2 = constant \tag{9.10}$$

When the spring is fully extended (x = A), the potential energy is maximum and the velocity is zero (KE=0). When the mass passes through the equilibrium position (x=0), the potential energy is zero and the kinetic energy is a maximum (v = $A\omega$).

9.3   Procedure

The apparatus is shown in Figure 9.1. First, the spring constant, k, will be measured using a force sensor and a motion detector. In the second part of the experiment, simple harmonic motion will be observed and the measured period compared to the value given by equation 9.5.

9.3.1 Measuring the Spring Constant k

- Remove the mass hanger, weights and spring from the force sensor. Press the 'tare' button on the force sensor. Replace the spring, mass hanger and weights.

- Open the experiment file 'k_measure.cap' A graph of force, position and force vs position will appear on the screen.

- Align the motion sensor directly under the mass hanger. **Avoid dropping the masses on the motion sensor.** Place .15 kg on the mass hanger (0.05kg) for a total mass of .2 kg. Secure the masses on the hanger with masking tape. Weigh the masses and hanger and record the value below. Attach the masses and hanger to the spring.

Mass _____

- Set the mass oscillating in the vertical direction and click 'record'. The mass should move only in the vertical direction and not side to side. Click 'stop' after the mass has oscillated through several oscillations.

- Fit the force vs position curve with a linear fit. The slope (m) of the fit is the spring constant. Why? Record the value of the spring constant below.

Spring constant (k) _____

- Save the data and fit as a '.cap' file and send the appropriately labeled file to your TA.

9.3.2 Measurement of the Period of Simple Harmonic Motion

- Close the k_measure file and open the file 'harmonic.cap' in Capstone. Plots of position, velocity and energies will appear. The energy plot on the right contains calculations of potential, kinetic and total energy. Verify the alignment of the motion sensor. Set the mass oscillating in the vertical direction and click 'record'. Collect data for five complete oscillations.

- Measure the period, T, using the 'smart tool' ($\overline{\diamond}$). Measure the time for 5 complete oscillations and find the period. Determine the angular frequency from $\omega = \frac{2\pi}{T}$. Record the values in the table. Fit the position curve with a 'sine' fit. Calculate the percentage difference between the fit values for the period, T. and value from the smart tool. Calculate the percentage difference between the angular frequency, ω, from the fit and the values measured with the 'smart tool'.

- Calculate the 'theoretical' value of ω from equation 9.5. Calculate the percentage difference between the experimentally determined fit value of ω and the 'theoretical' value.

	Period	ω
smart tool		
fit		
% diff		

ω from equation 9.5 _____

Percentage difference _____

- Click on the calculator icon (⊞) which will display the calculation for KE, PE or Etotal. Input the correct values for the mass constant (m), the force constant (k) and the position offset (offset). The offset is the distance from the face of the motion detector to the undisturbed (equilibrium) position of the mass. Close the calculator window by clicking on the calculator icon. Since the total energy (Etotal calculation) should be constant, adjust the offset value by small amounts (0.3 cm or less) to improve the total energy plot as necessary. Record the adjusted offset value below. Does the total energy plot show energy is conserved?

Offset _____

- Measure the amplitude of the oscillation on the position plot using the smart tool. Does the value of the energy calculated from equation 9.10 agree with the value of the total energy from the energy plot?

- Save the data and fit as a '.cap' file and send the appropriately labeled file to your TA.

9.3.3 Questions

1. Use dimensional analysis to show that the period, T, is proportional to $\sqrt{\frac{m}{k}}$

9.4 Conclusion

Write a detailed conclusion about what you have learned. Include all relevant numbers you have measured with errors. Sources of error should also be included.