Chapter 7

Momentum and Collisions

Name:________________ **Lab Partner:**____________ **Section:**____

7.1 Purpose

In this experiment, the conservation of linear momentum will be investigated. The application of momentum conservation to different types of collisions will be explored.

7.2 Introduction

Momentum, $\vec{p}$, is the product of mass and velocity

$$\vec{p} = m\vec{v} \tag{7.1}$$

Since velocity is a vector quantity, momentum is also a vector.

The total linear momentum in an isolated system always remains constant. A system is isolated in the sense that it doesn't interact with the rest of the world. Consider a group (system) of objects. Clearly if an **external** force acts on the objects, there is an acceleration, the velocity changes and thus the momentum changes. If only **internal** forces between the objects act, there is no net external force and the momentum of the system (group of particles) as a whole can not change. If no external forces act on a system of particles, the law of linear momentum conservation can be stated as:

- The total linear momentum of an isolated system remains constant (is conserved) if there are no **external** forces acting on the system.

This can be stated in mathematical form as:

$$\vec{p_f} = \vec{p_i} \tag{7.2}$$

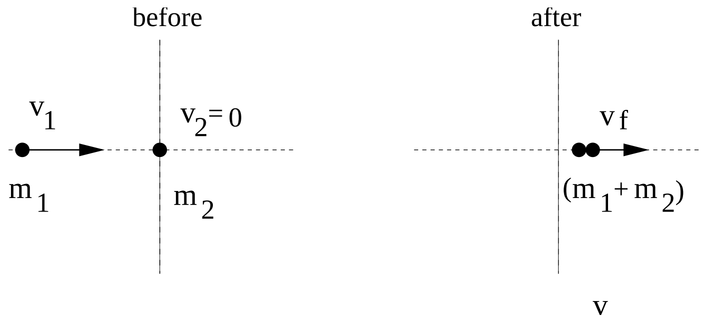

Figure 7.1: Schematic diagram of a totally inelastic collision.

where $\vec{p_f}$ is the final momentum of the whole system and $\vec{p_i}$ is the initial momentum of the whole system. Note that this is a vector equation. In Cartesian components (x-y), the x and y components are each conserved separately.

If two objects collide, momentum is conserved during the collision. However, we can further classify collisions depending on what happens to the kinetic energy ($\frac{1}{2}$mv^2) of the particles during the collision. Collisions can be classified as:

- An **elastic collision** is one in which the total kinetic energy is the same before and after the collision.
- An **inelastic collision** is a collision where total kinetic energy is **not** the same before and after the collision. If the objects stick together the collision is 'totally inelastic'.

The simplest of the collision situations to analyze is the totally inelastic case. m_1 enters from the left along the x axis with velocity $\vec{v}_1$ and strikes the target particle, m_2, at the origin. The target particle, m_2 is at rest. See Figure 7.1. After the collision, the two particles stick together into a single object of mass m_1 + m_2. The final velocity of m_1 + m_2 is $\vec{v}_f$. The final velocity, $\vec{v}_f$ must be along the x axis because of momentum conservation. The incoming momentum ($m_1 \cdot \vec{v}_1$) is all along the x axis i.e. there is no initial y component of momentum. Since momentum is conserved as a vector, there can be no component of the final momentum along the y axis.

Since the total momentum is conserved:

$$m_1 v_1 = (m_1 + m_2) v_f \tag{7.3}$$

and

$$v_f = \frac{m_1 v_1}{m_1 + m_2} \tag{7.4}$$

The fractional kinetic energy loss, $\frac{\Delta KE}{KE}$ is:

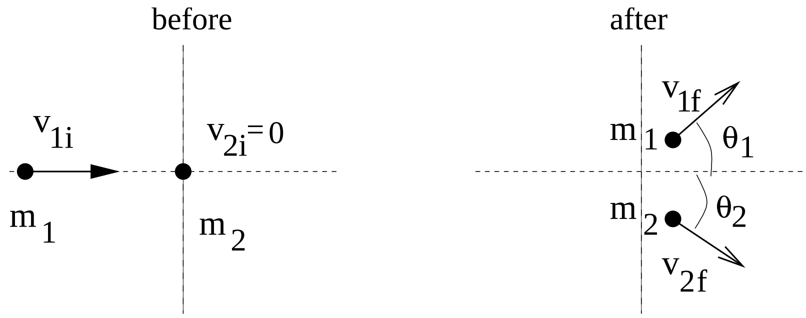

Figure 7.2: Schematic diagram of an elastic collision.

$$\frac{\Delta KE}{KE} = \frac{KE_i - KE_f}{KE_i} = \frac{\frac{1}{2}(m_1 v_1^2 - (m_1 + m_2) v_f^2)}{\frac{1}{2} m_1 v_1^2} \tag{7.5}$$

If v_f is substituted from equation 7.4 into this equation, the result is:

$$\frac{\Delta KE}{KE} = \frac{\frac{1}{2} v_1^2 (m_1 - \frac{m_1^2}{(m_1+m_2)})}{\frac{1}{2} m_1 v_1^2} = 1 - \frac{m_1}{m_1 + m_2} \tag{7.6}$$

This fractional kinetic energy loss is a measure of the in-elasticity of the collision.

An elastic collision requires more analysis, because of the outgoing particles can have different velocities which are not confined to the x axis. In an elastic collision, m_1 enters from the left traveling along the x axis as shown in figure 7.2. The second 'target mass', m_2 is stationary at the origin. After the collision, m_1 moves off with a velocity of $\vec{v}_{1f}$ in the first quadrant and m_2 moves off with a velocity of $\vec{v}_{2f}$ in the fourth quadrant. Each final velocity can have both x and y components. From conservation of momentum in the y direction we have:

$$m_1 v_{1f} sin\theta_1 = -m_2 v_{2f} sin\theta_2 \tag{7.7}$$

In the x direction we have:

$$m_1 v_{1i} = m_1 v_{1f} cos\theta_1 + m_2 v_{2f} cos\theta_2 \tag{7.8}$$

Since the collision is elastic, the total kinetic energy is conserved:

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \tag{7.9}$$

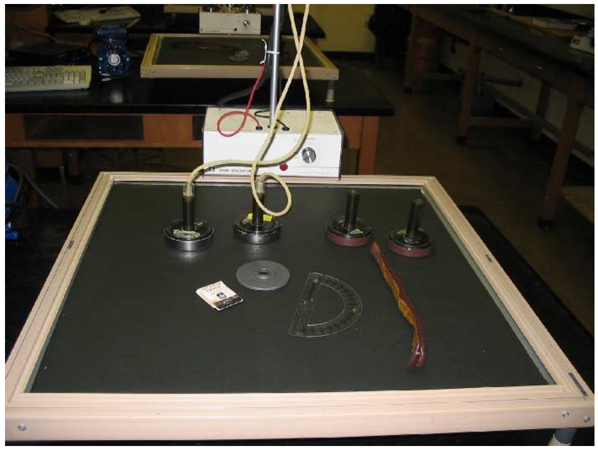

Figure 7.3: The air table with various components used in the experiment.

In addition to equations 7.7, 7.8, and 7.9, the components of the final velocities are related to the angles by:

$$\theta_1 = tan^{-1}(\frac{v_{1fy}}{v_{1fx}}) \tag{7.10}$$

and a similar equation for particle 2.

7.3 Procedure

An air table is used in this experiment. See Figure 7.3. Each puck is connected by a rubber hose to an air supply. The pucks 'float' on a cushion of air which significantly reduces friction. Inside of the rubber tube is a small flexible conductor which is connected to a spark timer. A high voltage pulse at regular time intervals (20 times per second or 20 Hz) causes a spark to burn a small hole in a piece of paper below the puck. Velcro can be placed around the pucks to cause them to stick together for an totally inelastic collision. Some of the pucks have magnets so they repel each other which makes an elastic collision situation. Lead collars can be placed on the pucks for collisions between unequal masses.

Special Cautions:

- **Do not touch the pucks or table when the spark timer is on.**
- **Be careful when removing the puck from the rubber hose to weigh the puck. Make sure the copper conductor and plug is replaced properly when re-connecting the rubber hose to the puck.**

7.3.1 Inelastic Collision - Unequal Masses

- Place a sheet of white paper on top of a graphite sheet on top of the table. Weigh the non-magnetic puck and its Velcro collar. Weigh the non-magnetic puck with the metal collar and its Velcro collar. Record both values below.

Mass of non-magnet puck ____________

Mass of non-magnet puck with metal collar ____________

- Set the spark timer to 20 Hz. Place the puck with the metal collar in the middle of the table and turn on the air supply. The puck should remain approximately motionless. If the puck moves, level the table by adjusting the legs of the table so the puck remain motionless.

- You may want to practice the next step without the spark timer a few times before attempting to take data with the spark timer.

- Turn on the power to the spark timer. Push the puck without the lead collar towards the stationary puck **with the insulating plastic tube**. At the same time one lab partner pushes the puck, the other partner should push and hold the trigger button on the spark timer. The pucks should stick together because of the Velcro collars and move off together. The pucks should not rotate after sticking together.

- Turn off the spark timer and remove the white paper. The 'dots' where the spark time burned the paper will appear on the side of the paper facing down.

- For each set of data before and after the collision, measure the dot spacings, Δx. Find the average dot spacing, $\Delta\bar{x}$. Calculate the velocity before and after the collision using v $= \frac{\Delta\bar{x}}{\Delta t}$ where $\Delta t = \frac{1}{f} = \frac{1}{20}\text{s} = 0.05\text{s}$. Calculate the momentum before and after the collision. Calculate the kinetic energy before and after the collision. Record you numbers in the table below.

	$\Delta\bar{x}$	Velocity	Momentum	Kinetic Energy
Before Collision				
After Collision				

- Is momentum conserved during the collision? Is kinetic energy conserved?

7.3.2 Elastic Collision with Equal Masses

- Reuse the white paper by turning it over or using a different region of the paper. Weigh two magnetic pucks and record the values below The spark timer should remain set at 20 Hz.

Mass of magnetic projectile (m_1) puck ____________
Mass of magnetic target (m_2) puck ____________

- Verify the table is level as described in the inelastic collision. Turn on the spark timer.
- Place the target puck in the center of the table. Trigger the spark timer as the the other puck is pushed towards the stationary puck with an insulating plastic tube. The collision should not be head-on (1 dimensional).
- Remove the paper from the table. Take the (negative) x axis along the track of the incoming puck and the origin at the point of collision. Find the average dot spacing, $\Delta\bar{x}$, for each track (incoming puck track and the two outgoing puck tracks). Using the average dot spacing and the 20 Hz frequency, determine the velocities of each track. Using a protractor, determine the angles of the outgoing tracks with respect to the positive x axis. Calculate the momentum components of each of the tracks and show that momentum is conserved in the collision. Record your data in the table.

Track	$\Delta\bar{x}$	v	P	Θ	P_x	P_y
Incoming						
Target						
Projectile						

- Using the magnitude of the velocity for each track show that kinetic energy is conserved in the collision.

7.3.3 Elastic Collision with Unequal Masses

- Repeat the elastic collision procedure using the metal collar on the stationary puck.

Mass of magnetic projectile (m_1) puck ____________
Mass of magnetic target (m_2) puck with metal collar ____________

Track	$\Delta\bar{x}$	v	P	Θ	P_x	P_y
Incoming						
Target						
Projectile						

- Using the magnitude of the velocity for each track show that kinetic energy is conserved in the collision.

7.3.4 Questions

1. For the inelastic collision, does the calculated fractional kinetic energy change (equation 7.6) agree with the measured value for the fractional kinetic energy change?

2. If your data shows that kinetic energy is not conserved in either of the elastic collision procedures, what sources of error might account for the discrepancy?

7.4 Conclusion

Write a detailed conclusion about what you have learned. Include all relevant numbers you have measured with errors. Sources of error should also be included.