Chapter 4

Newton's 2^{nd} Law

Name:______________ **Lab Partner:**___________ **Section:**____

4.1 Purpose

In this experiment, Newton's 2^{nd} law will be investigated.

4.2 Introduction

How does an object change its motion when a force is applied? A force is simply a push or a pull. Is an object's velocity related to the applied force or does the force just change the object's velocity? What role does mass play? Isaac Newton's 2^{nd} law answers all of these questions about the dynamics of an object's motion. Newton's 2^{nd} law can be stated as:

$$\vec{F}_{net} = m\vec{a}$$

where $\vec{F}_{net}$ is the **net** force applied to an object of mass m and $\vec{a}$ is the acceleration of the object. Note this is a vector equation.

4.2.1 Inclined Plane

In the first part of the experiment, we will measure the motion and force on a cart as it is pulled up an inclined plane by a mass. A schematic of the configuration is shown in Figure 4.1.

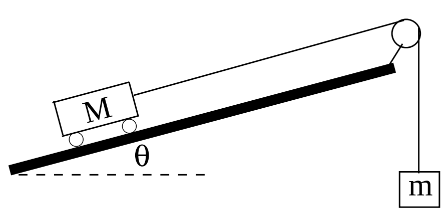

Figure 4.1: Schematic of the inclined plane

To analyze the situation, consider the free body diagrams of the cart with mass M and the hanging weight with mass m shown in Figure 4.2. For the hanging mass m in the vertical

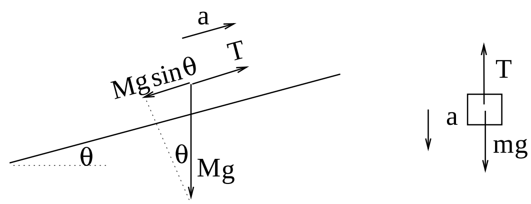

Figure 4.2: Free body diagrams for M and m

direction, Newton's 2^{nd} law gives:

$$F_{net} = ma$$

$$F_{net} = T - mg = m(-a)$$

and

$$T - mg = -ma \tag{4.1}$$

Where T is the tension in the string, m is the mass, g is the acceleration of gravity and a is the acceleration. Note that the acceleration is in the negative direction. For the cart with mass M, consider the forces **parallel** to the inclined plane. Newton's 2^{nd} law gives:

$$F_{net} = T - Mgsin\theta = Ma$$

and

$$T - Mgsin\theta = Ma \tag{4.2}$$

where T is the tension in the string which is the same tension for the hanging mass, a is the acceleration which is the same for the hanging mass, θ is the angle the inclined plane makes with the horizontal direction, and M is the mass.

Equation 4.1 and Equation 4.2 are two simultaneous equations for a and T. Solving for a by eliminating T gives:

$$a = \frac{mg - Mgsin\theta}{M + m} \tag{4.3}$$

T can then be found by substituting a into Equation 4.1 or Equation 4.2

4.2.2 Atwood Machine

The physical apparatus for the second part of the experiment is an 'Atwood machine'. The 'Atwood machine' consists of two unequal masses connected by a string over a pulley. In figure 4.3, m_1 is greater than m_2 so m_2 moves up and m_1 moves down.

To understand the Atwood machine using Newton's 2^{nd} law, consider the motion of the two masses moving under the influence of gravity. The string connecting the masses has a tension, T, which in both cases is directed upward. Both masses have an acceleration of equal magnitude a but m_2 is accelerating upwards and m_1 is accelerating downward. 'Free

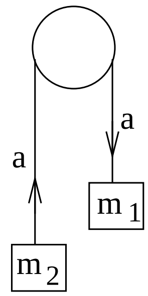

Figure 4.3: Basic schematic of the Atwood machine

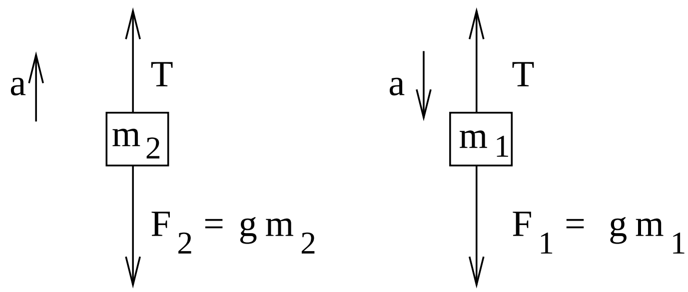

Figure 4.4: Free body diagrams for m_1 and m_2

body' diagrams which show the forces acting on the individual masses and their accelerations are shown in Figure 4.4.

For m_1, Newton's 2^{nd} law gives:

$$T - gm_1 = -am_1 \tag{4.4}$$

Applying Newton's 2^{nd} law to m_2 yields:

$$T - gm_2 = am_2 \tag{4.5}$$

Note that the signs for the acceleration, tension, and gravitational force have been explicitly included in equations equation 4.4 and equation 4.5. These two simultaneous equations can be solved for the acceleration and tension. The acceleration, a, is given by:

$$a = \frac{(m_1 - m_2)g}{(m_1 + m_2)} \tag{4.6}$$

4.3 Procedure

First we will measure the acceleration and tension in the inclined plane configuration and then measure the acceleration of an Atwood machine.

4.3.1 Inclined Plane

The 'dynamic cart' system is shown in Figure 4.5. The pulley at the end of the track is a so-called smart pulley. The smart pulley consists of a small plastic wheel and a photo-gate.

The pulley wheel is designed to have a small mass and low friction in the bearings. The photo-gate has an infrared beam on one side which is detected on the opposite side. The spokes of the wheel allow the beam to pass through the open region between the spokes or block the beam. The motion of the wheel can then be detected as the mass falls. The force sensor is attached to the cart. A string is attached to the force sensor which is positioned over the smart pulley and is attached to the falling mass. The end of the track with the smart pulley is elevated by a wood block.

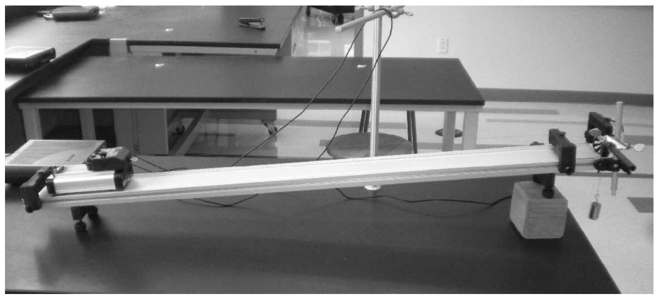

Figure 4.5: Apparatus for the inclined plane experiment

- Remove the wood block from under the end of the track with the smart pulley. Verify the track is level. The cart should not move either direction if the track is level. The feet of the track can be adjusted if necessary to level the track. Measure the distance between the feet, d, of the track. Measure the thickness of the wood block, t. Record both values below.

Distance between feet of the track (d) ____________
Thickness of wood block (t) ____________

- The elevation angle of the track, θ, is $\sin^{-1}$(t/d). Calculate θ and record it below.

Elevation angle (θ) ____________

- Weigh the glider and attached force sensor. Record the value below.

Mass of cart and force sensor (M) ____________

- Open the file '2nd_law' in Capstone. A graph of force vs time and velocity vs time will appear.

- Place the block under the feet of the track nearest the smart pulley. Place the cart on the track. Without the string attached, press the 'Tare' button on the force sensor. Attach the string to the force sensor, over the smart pulley and to the mass hanger with 0.1 kg of mass (a 50 grams mass and a 50 gram hanger). Hold the cart at the end of the track away from the smart pulley with your hand. The string must be on the pulley of the smart pulley.

- Click 'record' and then release the cart. Click 'stop' after the cart reaches the end of the track.

- Identify and highlight the region on the velocity vs time graph where the cart was moving. Fit this region with a linear fit. The slope of the line (m) is the acceleration (a).

- Identify and highlight the region of the force vs time graph where the cart was moving. The statistics function mean (Σ) for the force during this time is the tension in the string (T). Record the values as trial 1 in the data table. (Note that the force value is negative because of the orientation of the force sensor. In the coordinate system defined in the introduction, T is positive.)

- Repeat the measurement two more times for a total of 3 measurements. Record the data in the data table. Calculate and record the mean of the acceleration and tension.

Trial	Acceleration (a)	Tension (T)
1		
2		
3		
Sum		

Table 4.1: Data table for the inclined plane data

Mean of acceleration (a) ____________

Mean of Tension (T) ____________

- Calculate the acceleration from Equation 4.3 and find the tension using either Equation 4.1 or Equation 4.2. Record the values below.

Calculated acceleration ____________

Calculated Tension ____________

- Find the percent error between the calculated and experimental values (mean) for a and T

% error in a ____________

%error in T ____________

4.3.2 Atwood Machine

The two masses of .200 kg and .205 kg are hung over the 'smart pulley' which detect the motion of the string as the masses move (See figure 4.6.

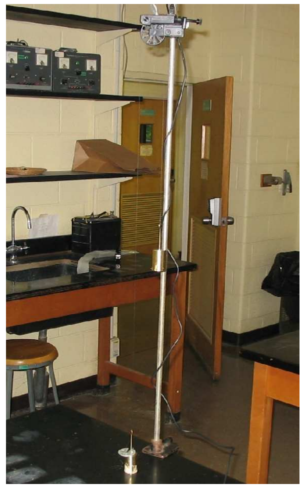

Figure 4.6: The Atwood machine. The smart pulley is mounted at the top. The two masses connect by a string over the smart pulley

- The two masses hanging from the string are 0.200 kg (brass) and 0.205 kg (stainless steel). Do not weigh the masses since they are difficult to replace on the string. Record the values of the masses.

Mass of m_1 ____________
Mass of m_2 ____________

- Open the file 'atwood.cap' in Capstone. Two plots will appear on the screen. One plot is velocity vs time. The other plot is position vs. time.
- Hold the smaller brass mass near the tabletop. It should be positioned so it does not swing when released. Release the mass from rest and 'click' the record button. 'Click' stop just before the larger mass hits the table.
- Highlight the time region of interest on the position vs. time plot. Fit this region with a quadratic fit. The quadratic term in the fit ('A') is $\frac{1}{2}$a.

Acceleration from quadratic fit of position vs time graph ____________

- Highlight the time region of interest on the velocity vs time plot. Fit the region of interest with a linear fit. The slope of this line is the acceleration, a, of the masses. Record this value of the acceleration in the data table below as trial 1.

Mean of 5 trials ____________

- Repeat the measurement 4 more times for five total measurements. Using the value of the acceleration from the five fits to the velocity vs time graph, determine the mean of your five measurements.

Trial	Acceleration
1	
2	
3	
4	
5	
Sum	

- Calculate the percentage error between the mean value of the acceleration from the data table and the 'theoretical' value calculated from Equation 4.6.

Acceleration from Equation 4.6 ______________

Percentage error ______________

- Save one example set of data (.cap file) from the five measurements and submit them to your TA by email attachment. (Be sure to label the the file with your name, your lab partner's name, and the lab name.)

4.3.3 Questions

1. Solve equation 4.1 and equation 4.2 for the acceleration of the cart (equation 4.3) and the tension in the string.

2. Solve equation 4.4 and equation 4.5 for the acceleration of the masses (equation 4.6) and the tension in the string.

3. The mass of the pulley and the friction in the bearings of the pulley and cart are small. Is your data consistent with the pulley or cart having a small but nonzero effect on the motion?

4.4 Conclusion

Write a detailed conclusion about what you have learned. Include all relevant numbers you have measured with errors. Sources of error should also be included.