

# Chapter 3

## Vectors

Name: \_\_\_\_\_ Lab Partner: \_\_\_\_\_ Section: \_\_\_\_\_

### 3.1 Purpose

In this experiment vector addition, resolution of vectors into components, force, and equilibrium will be explored.

### 3.2 Introduction

A vector is probably the most frequently used entity in physics to characterize space. It can represent the spatial behavior of many physical quantities such as forces, velocities, accelerations, and electric fields.

A vector is a quantity that has both magnitude and direction. A quantity which does not have direction is called a scalar quantity. In order to distinguish vectors from scalars, vectors are identified with either bold face or an arrow on the top of the letter:  $\mathbf{V}$  or  $\vec{V}$ .

A vector can be represented by an arrow with length proportional to the magnitude. The direction of the arrow gives the direction of a vector. Two or more vectors can be added together. To add two vectors **graphically**, the tail of the second vector is placed at the head of the first vector. A vector from the tail of the first vector to the head of the second vector is the sum of the two vectors (see Figure 3.1).

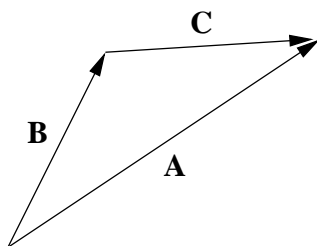


Figure 3.1: Graphical representation of vector addition,  $\vec{A} = \vec{B} + \vec{C}$ .

A vector treated as a geometrical object is **independent** of any coordinate system. Once a coordinate system is introduced, the vector can be represented by a set of coordinates. For a n dimensional space, a vector is an array of n numbers. This array of numbers is the coordinates of the head of the vector with the tail of the vector at the origin. For example, consider a vector  $\vec{F}$  at the origin (see Figure 3.2). The head of the vector has coordinates  $(F_x, F_y)$ , so  $\vec{F} \rightarrow (F_x, F_y)$ . Using F for the magnitude of  $\vec{F}$  and trigonometry the components of the vector are:

$$F_x = F \cos \theta \quad F_y = F \sin \theta \quad (3.1)$$

where  $\theta$  is the angle  $\vec{F}$  makes with the positive x axis.  $(F_x, F_y)$  are known as the (Cartesian) *components* of  $\vec{F}$ . If the components of a vector are known, the magnitude of the vector can be found from the Pythagorean theorem ( $|F| = \sqrt{F_x^2 + F_y^2}$ ) and the direction from  $\theta = \tan^{-1}(\frac{F_y}{F_x})$ . The choice between using  $\vec{F}$  or  $(F_x, F_y)$  is essentially a choice between geometric and algebraic representation.

In this lab you will study two dimensional static forces using the force table where  $\sum \vec{F}_i = 0$ . Stated another way, the vector resolution of the forces must sum to zero under static equilibrium. (Later when dynamics are studied, we will see this is a consequence of Newton's second law.) Given this condition - that the algebraic sum of the x and y components must individually be zero - some components will be positive and some will be negative along a particular axis.

The force table consists of a circular metal disk having a calibrated angular scale (see Figure 3.3). Three masses,  $m_i$ , are suspended from the disk's rim with strings. The three strings are tied together at the center of the disk. The masses and/or angular positions of the strings are adjusted until the three mass + string system is in static equilibrium and doesn't move. The force acting along the string is proportional to the mass hung from that string:

$$|F_i| = m_i g \quad (3.2)$$

where  $g = 9.8 \text{ m/s}^2$ .

The following is an example of addition of vector forces. Briefly, the force vectors  $(\vec{F}_a, \vec{F}_b, \vec{F}_c)$  will be resolved into their respective vector components along the x and y-axis and a resultant force vector will be obtained by algebraically adding the components of the respective axes. The equilibrium force is a force vector that statically balances the resultant force. Consider three vectors  $\vec{F}_a$ ,  $\vec{F}_b$ , and  $\vec{F}_c$  with all of the angles measured with respect to the x-axis. Each of the vector components will need to be resolved and summed to find the resultant.

$$\begin{aligned} \vec{F}_a &= 150 \text{ N at } 0^\circ \\ \vec{F}_b &= 110 \text{ N at } 70^\circ \\ \vec{F}_c &= 250 \text{ N at } 135^\circ \end{aligned}$$

with all the angles measured from the positive x-axis.

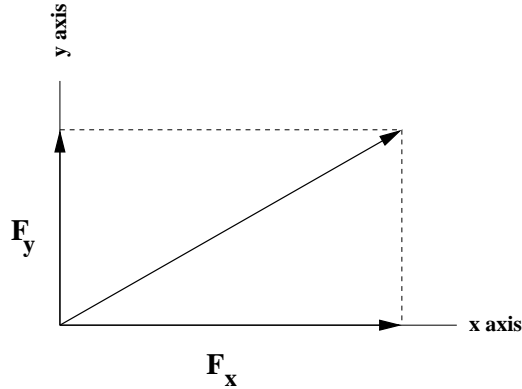


Figure 3.2: The resolution of a vector  $\vec{F}$  into components in a Cartesian (x-y) coordinate system.

$$\sum F_x = 150 \text{ N} \cos 0^\circ + 110 \text{ N} \cos 70^\circ + 250 \text{ N} \cos 135^\circ = 10.84 \text{ N}$$

$$\sum F_y = 150 \text{ N} \sin 0^\circ + 110 \text{ N} \sin 70^\circ + 250 \text{ N} \sin 135^\circ = 280.15 \text{ N}$$

The magnitude of the resultant is then:

$$R = \sqrt{(10.84 \text{ N})^2 + (280.15 \text{ N})^2} = 280.35 \text{ N}.$$

The direction of this vector magnitude is found by taking the arc-tangent of the ratio of the y-axis components and the x-axis components.

$$\arctan \frac{R_y}{R_x} = \arctan \frac{280.15}{10.84} = 87.78^\circ$$

This angle is taken from the x-axis as its point of origin. This is known as the resultant vector,  $\vec{R}$ . The equilibrant vector has the same magnitude as the resultant but opposite direction. Thus,  $\theta_{\text{equilibrant}} = 267.78^\circ$ .

### 3.3 Procedure

A picture of the force table is shown in Figure 3.3. Three strings are attached to the a center ring. A pin holds the center ring in position while angles and weights are being adjusted. The pin can be then be removed to check for static equilibrium. Each string is placed over a pulley and connected to a weight hanger. The pulley should be mounted perpendicular to the edge of the table and the string should be positioned in the slot of the pulley wheel. A weight hanger is attached the end of the string. The mass of the weight hanger should always be included in the total mass being considered.



Figure 3.3: The force table used for the experiment

### 3.3.1 Two Masses and Angles Fixed

- Hang 0.2kg of mass (including the mass hanger) at  $0^\circ$ . When placing masses on the force table always verify the pulley is at the correct angle and the string is properly positioned over the pulley wheel. Place another 0.2kg of mass (including the mass hanger) at  $240^\circ$ .
- Using the graphical method, determine the resultant of the two forces. The equilibrant vector force is the negative (vector rotated  $180^\circ$ ) of the resultant vector. Be sure to draw a proper graph when using the graphical method. Include the properly labeled and referenced graph with your report. Graph paper is provided at the end of this chapter.
- Place a pulley at the equilibrant vector position with the proper mass to produce the equilibrant force. Is the system in static equilibrium?
- Using the component method, calculate the resultant and equilibrant vector for the two given masses and angles in the table below.

	Magnitude	Angle	x-component	y-component
Force 1		$0^\circ$		
Force 2		$240^\circ$		
resultant				
equilibrant				

- Which method (graphical or component) gives the most accurate result?

### 3.3.2 Masses Fixed with Angles Varied

- Select two equal masses ( $m_a$  and  $m_b$ ) and another mass ( $m_c$ ) which is  $\frac{1}{2}$  of  $m_a$  and  $m_b$ , i.e.  $m_c = \frac{1}{2}m_a$  and  $m_c = \frac{1}{2}m_b$ . Record the values of the masses and the forces they produce.

$m_a$  \_\_\_\_\_       $F_a$  \_\_\_\_\_  
 $m_b$  \_\_\_\_\_       $F_b$  \_\_\_\_\_  
 $m_c$  \_\_\_\_\_       $F_c$  \_\_\_\_\_

- To simplify the later calculation, place  $m_a$  at  $0^\circ$ . Obtain static equilibrium by varying the angles. Record the angles of  $m_b$  and  $m_c$ .

$\theta_b$  \_\_\_\_\_  
 $\theta_c$  \_\_\_\_\_

- Using the graphical method, show the system is in static equilibrium. Include the properly labeled and referenced graph with your report. Graph paper is provided at the end of this chapter.
- Using the component method, show the system is in static equilibrium.

Vector	Magnitude	Angle	x-component	y-component
Vector A		$0^\circ$		
Vector B				
vector C				
resultant				

- Keeping the masses and magnitudes of the forces fixed, rotate each vector by  $30^\circ$  i.e., add  $30^\circ$  to each angle. Is the system still in static equilibrium? Using the component method, calculate the components of each vector at its new angle. Does the component method still show the system to be in equilibrium?

Vector	Magnitude	Angle	x-component	y-component
Vector A		$30^\circ$		
Vector B				
vector C				
resultant				
equilibrant				

### 3.3.3 Questions

1. Are the vector components of a force equivalent to the force itself? Is a complete set of components together with knowledge of the coordinate system equivalent to the force itself?
2. Does the order in which vector components are added affect the resultant of the vectors?
3. Which of the following quantities are vectors: speed, velocity, mass, weight, force, and volume?

## 3.4 Conclusion

Write a detailed conclusion about what you have learned. Include all relevant numbers you have measured with errors. Sources of error should also be included.

