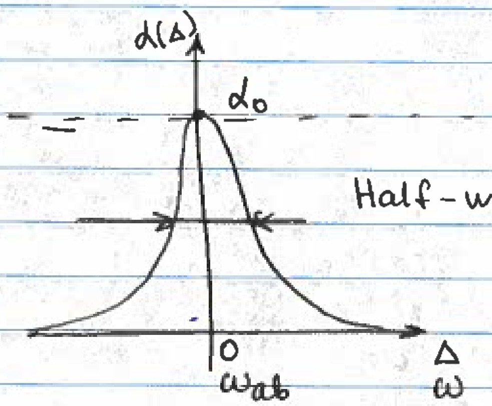
Inhomogeneous broadening (due to atomic motion, aka Doppler broadening)

Previously, we have calculated the absorption coefficient for a two-level system

$$\alpha(\Delta) = \alpha_0 \frac{L(\Delta)}{1 + I/I_s \cdot L(\Delta)} \qquad L(\Delta) = \frac{\gamma_{ab}^2}{\gamma_{ab}^2 + \Delta^2}$$

$$\alpha(\Delta) = \alpha_0 \frac{\gamma_{ab}^2}{\gamma_{ab}^2 + \Delta^2 + I/I_s\, \gamma_{ab}^2} = \alpha_0 \frac{\gamma_{ab}^2}{\Delta^2 + \gamma_{ab}^2 (1 + I/I_s)}$$

$$\Delta = \omega - \omega_{ab}$$



Half-width half-maximum

$$\Delta_{HWHM} = \gamma_{ab} \underbrace{\sqrt{1 + I/I_s}}_{\text{power broadening}}$$

This is a <u>homogeneous</u> linewidth: it is the same for all atoms (all atoms are identical)

<u>Inhomogenious</u> <u>broadening</u> arises from different atoms under different conditions within the same ensemble, exhibiting different responces.

Example: thermal motion of an atomic gas

Maxwell distribution:

$$dn(v_z) = n(v_z)\, dv_z = \underbrace{\frac{N}{V}}_{\text{atomic density}} e^{-v_z^2/v_T^2} \frac{1}{\sqrt{\pi}\, v_T} dv_z$$

$$v_T = \sqrt{\frac{8 k_B T}{\pi m}}$$

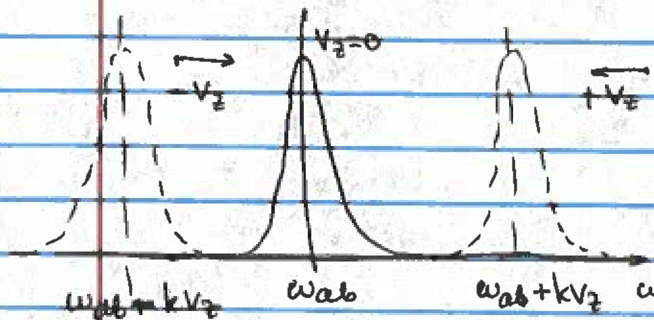
k_B - Boltzmann constant

Why atomic motion matters? Doppler effect

$\omega_{ab} \longrightarrow \omega_{ab} + kv_z$ $\quad k = \frac{2\pi}{\lambda}$

stationary atom $\quad$ moving atom

$\Delta = \omega - \omega_{ab} \longrightarrow \Delta = \omega - (\omega_{ab} + kv_z)$



absorption resonance occurs at different frequency for atoms in different velocity groups

Thermal ensembles

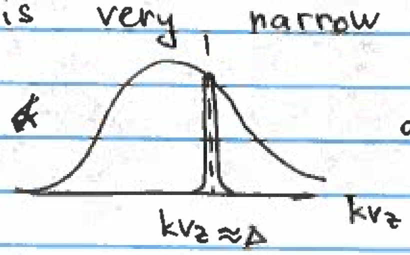
$$\alpha(\Delta) = \int_{-\infty}^{+\infty} \alpha(\Delta, kv_z)\, n(v_z)\, dv_z =$$

$$= \alpha_0 \int_{-\infty}^{+\infty} \frac{\gamma_{ab}^2}{(\Delta + kv_z)^2 + \gamma_{ab}^2(1 + I/I_s)} \frac{N}{V} \frac{1}{\sqrt{\pi}\, k v_T} e^{-(kv_z)^2/(kv_T)^2} dv_z$$

Voight profile

Two limiting cases: cold atoms $kv_T \ll \gamma_{ab}$
negligible inhomogeneous broadening

Hot atoms $kv_T \gg \gamma_{ab}$: homogeneous profile is very narrow $\longrightarrow$ delta function at $\Delta = kv_z$



$$\alpha_D(\Delta) = \alpha_0 \frac{N}{V} \frac{\gamma_{ab}}{\sqrt{\pi}\, k v_T} e^{-\Delta^2/(kv_T)^2}$$

$$\Delta_D = kv_T$$

$$\alpha_D(\Delta) = \alpha_0^{(Dop)} e^{-\Delta^2/\Delta_D^2}$$

$$\alpha_0^{(Dop)} \sim \alpha_0 \frac{\gamma_{ab}}{\Delta_D}$$

Much less absorption compare to the cold atomic ensemble of the same density.

In general, Voight profile (a convolution of Lorenzian and Gaussian profiles) can only be evaluated numerically. However, there is a hack to get an approximate analytical solution:

$$\alpha(\Delta) = \int \alpha^{(\text{hom})}(\Delta - kv_z)\, \underbrace{e^{-(kv_z)^2/\Delta_D^2} \frac{d(kz)}{\sqrt{\pi}\,\Delta_D}}$$

replace with Lorenzian of the same width

$$\frac{1}{\pi} \frac{\Delta_D^2}{(kv_z)^2 + \Delta_D^2} \frac{d(kv_z)}{\Delta_D}$$

Then this integral can be evaluated analytically via residue theorem.

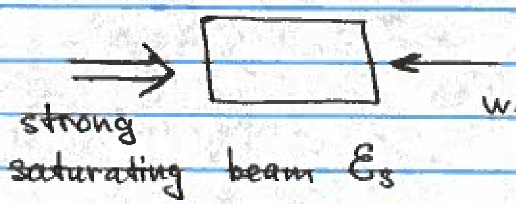
Saturation spectroscopy

Main goal → to enable measurements of homogeneously-broadened absorption profile in an inhomogeneously-broadened atomic medium

(For Rb atoms at room temperature

$\gamma_{ab} \simeq 6\ \text{MHz}$ $\qquad \Delta_D \simeq 300\ \text{MHz}$

Two counter-propagating beams

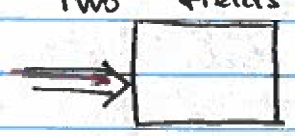


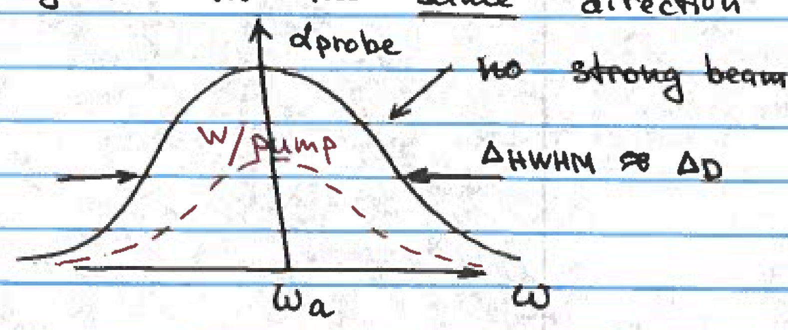
$$E_{total} = \mathcal{E}_s e^{ikz - i\omega t} + \mathcal{E}_p e^{-ikz - i\omega t} + c.c.$$

The strong beam is strong enough so that it saturates the absorption ~~profile~~ for the velocity group it interacts with

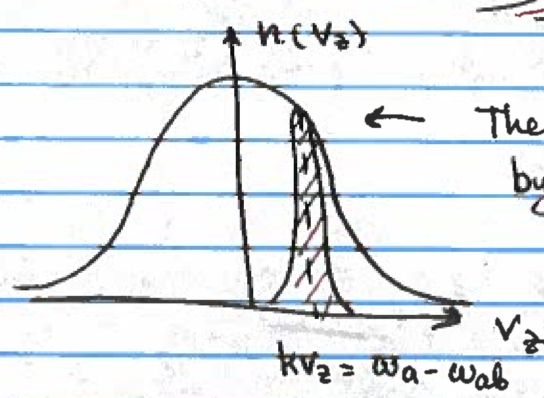
~~Z~~ The weak field is weak enough to not saturate atoms.

Two fields propagate in the same direction





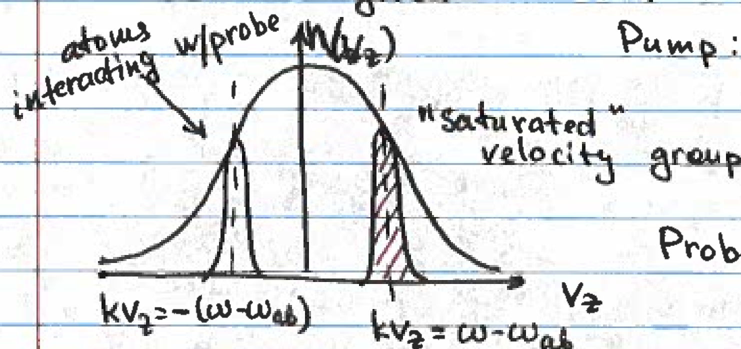
For a given ω



← These atoms are saturated by the strong pump field. Since these are the same atoms that interact with the probe, the probe absorption is reduced for any laser detuning.

Counter-propagating field

For a given $\omega \neq \omega_{ab}$

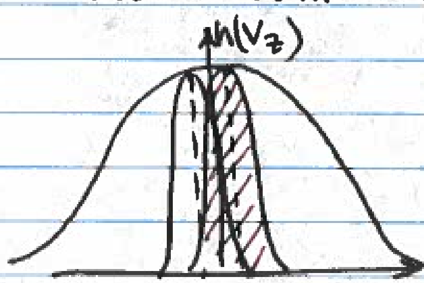


Pump: $kv_z^{(pump)} = \omega - \omega_{ab}$

Probe: $-kv_z^{(probe)} = \omega - \omega_{ab}$

Since probe and pump interacting with different atoms, the presence of the pump does not affects the absorption for the probe.

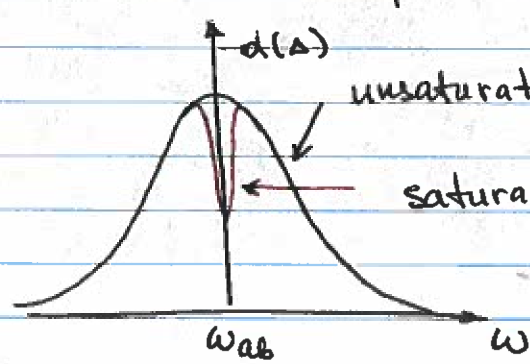
However! if $|\omega - \omega_{ab}| \lesssim \gamma_{ab}^{(hom)}$, same atoms "see" both optical field, so some of the atoms have reduced absorption.



Best overlap $V_z = 0$

or when $\omega = \omega_{ab}$

Probe absorption



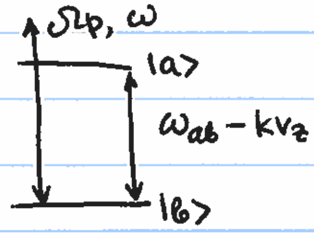
unsaturated probe absorption

saturation absorption ~~peak~~ dip

width $\sim \gamma_{ab}^{(hom)}$

$\omega = \omega_{ab}$

How to calculate sat. spec. resonances

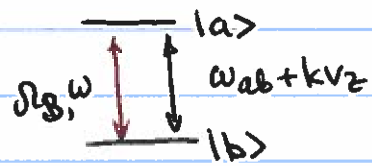


$$d_p = \frac{k}{2}\, \mathrm{Im}\left(\frac{P_p}{\varepsilon_0 \mathcal{E}_p}\right)$$

$$P_p = \int n(v_z)\, \wp_{ab}\, \rho_{ab}(\Delta + kv_z)\, dv_z$$

$$\rho_{ab} = i\Omega_p \frac{(\rho_{aa} - \rho_{bb})}{\gamma_{ab} + i(\Delta + kv_z)} \qquad \boxed{\Delta = \omega - \omega_{ab}}$$

We will assume that the population difference is determined by the <u>strong</u> field



$$\rho_{aa}^{(S)} - \rho_{bb}^{(S)} = \frac{1}{1 + S\cdot L(\Delta - kv_z)}$$

S – saturation parameter for the pump field

$$\frac{1}{1+S\cdot L(\Delta - kv_z)} = \frac{\gamma_{ab}^2 + (\Delta - kv_z)^2}{(\Delta - kv_z)^2 + \gamma_{ab}^2(1+S)} =$$

$$= 1 - \frac{\gamma_{ab}^2 S}{(\Delta - kv_z)^2 + \gamma_{ab}^2(1+S)}$$

$$\chi_p(\Delta) = i\frac{|\wp_{ab}|^2}{\hbar\varepsilon_0}\int d(kv_z)\, n(v_z) \frac{(\rho_{aa}^{(S)} - \rho_{bb}^{(S)})}{\gamma_{ab} + i(\Delta + kv_z)} =$$

$$= i\frac{|\wp_{ab}|^2}{\hbar\varepsilon_0}\int d(kv_z)\, n(v_z) \frac{1}{\gamma_{ab} + i(\Delta + kv_z)}\left[1 - \frac{\gamma_{ab}^2 \cdot S}{(\Delta - kv_z)^2 + \gamma_{ab}^2(1+S)}\right]$$

The first term — "regular" unsaturated inhomogeneously - broadened ~~absorp~~ susceptibility $\chi_p^{(0)}$ &

~~The~~ second term → saturation effect of the strong field

$$d_p(\Delta) = \frac{k}{2}\,\mathrm{Im}\,(\chi_p(\Delta)) = d_p^{(0)} - \underbrace{\frac{|\wp_{ab}|^2}{\varepsilon_0\hbar}\int n(v_z)\, d(kv_z) \times \ldots}_{\text{interesting part}}$$

~~one more assumption : since~~

$$\times \frac{\gamma_{ab}}{\gamma_{ab}^2 + (\Delta + kv_z)^2} \cdot \frac{S\gamma_{ab}^2}{\gamma_{ab}^2(1+S) + (\Delta - kv_z)^2}$$

To get the analitycal solution we assume that saturation is not strong. Also, we can see that the expression only has non-vanishing contribution if $v_z \approx 0$ so $n(v_z) \approx n(0)$ does not change anything

$$d_p(\Delta) = d_p^{(0)} - \frac{|\wp_{ab}^{*}|^2}{\varepsilon_0\hbar} n(0) \int \frac{\gamma_{ab}}{\gamma_{ab}^2 + (\Delta + kv_z)^2} \, \frac{S\gamma_{ab}^2}{\gamma_{ab}^2 + (\Delta - kv_z)^2} \, d(kv_z)$$

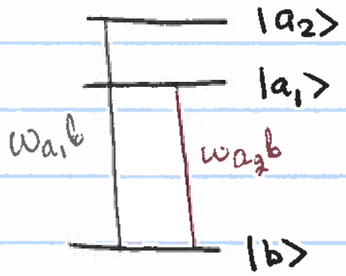
$$= d_p^{(0)} - \frac{\pi}{2} \frac{n(0)|\wp_{ab}|^2}{\varepsilon_0\hbar} \frac{S}{\gamma_{ab}^2 + \Delta^2}$$

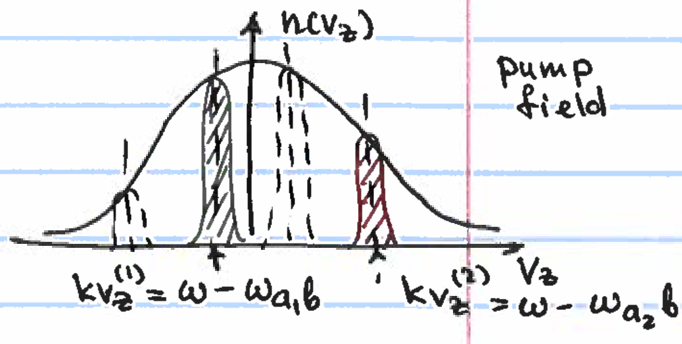
$$\left[\int_{-\infty}^{+\infty} \frac{1}{1+(x-a)^2} \frac{1}{1+(x+a)^2} dx = \frac{\pi}{2(1+a^2)} \right]$$

Second term: reduced apsorption (Lamb dip) or peak in transmission @ $\Delta = 0$, $\omega = \omega_{ab}$ width γ_{ab} (for weak saturation) also subject of the power broadening

Cross-over resonances

appear when more than one atomic resonance is inside the Doppler width





When the probe field "sees" saturated atoms?

$\omega = \omega_{a_1b}$ or $\omega = \omega_{a_2b}$

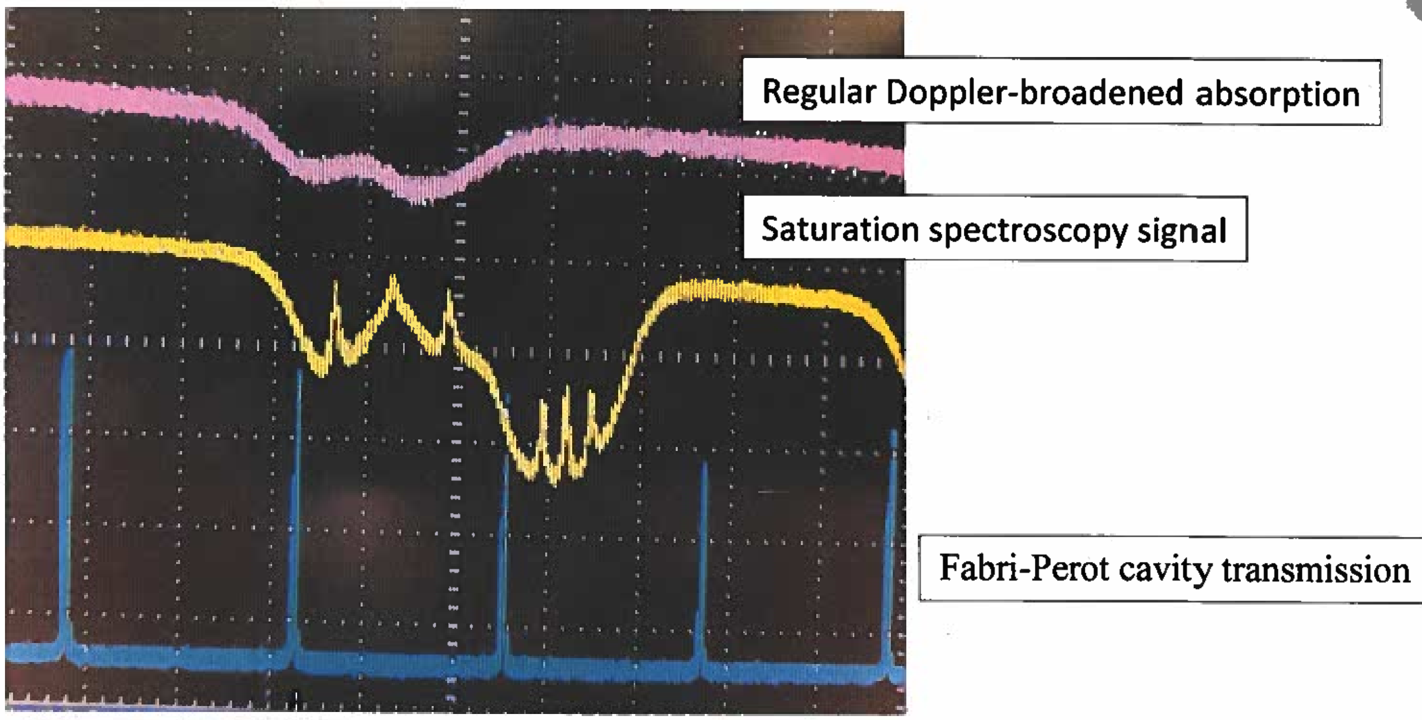
(the probe and pump interact with atoms saturated at the same transition)

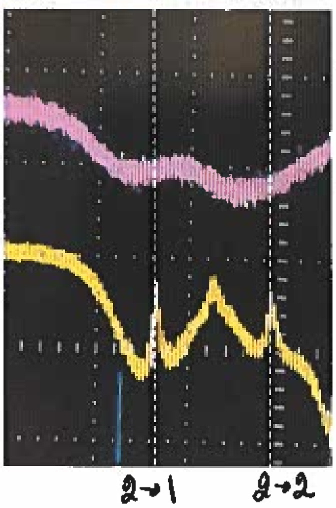
one more option: $\omega = \underbrace{\omega_{a_1b} + kv_z}_{\text{pump}} = \underbrace{\omega_{a_2b} - kv_z}_{\text{probe}}$

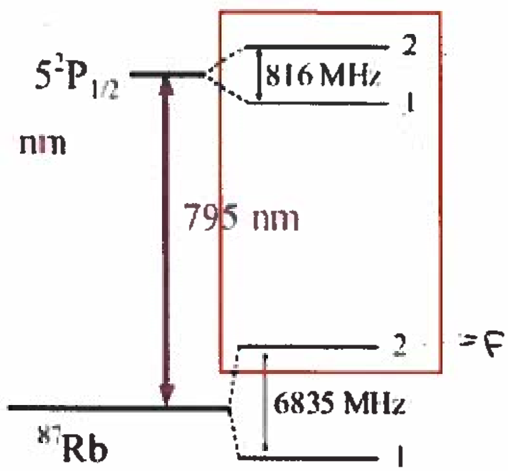
Possible for $\omega = \dfrac{\omega_{a_1b} + \omega_{a_2b}}{2}$

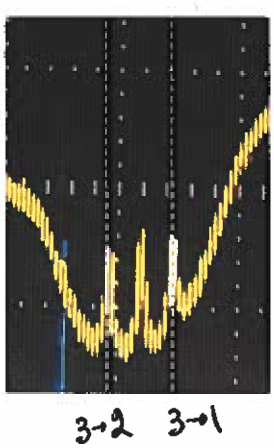
cross-over resonance

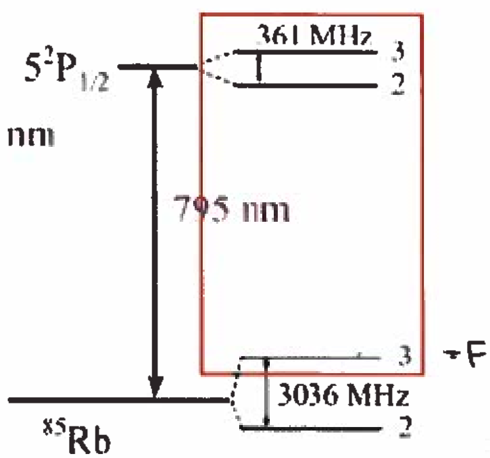
Rb D1 line


Regular Doppler-broadened absorption
Saturation spectroscopy signal
Fabri-Perot cavity transmission

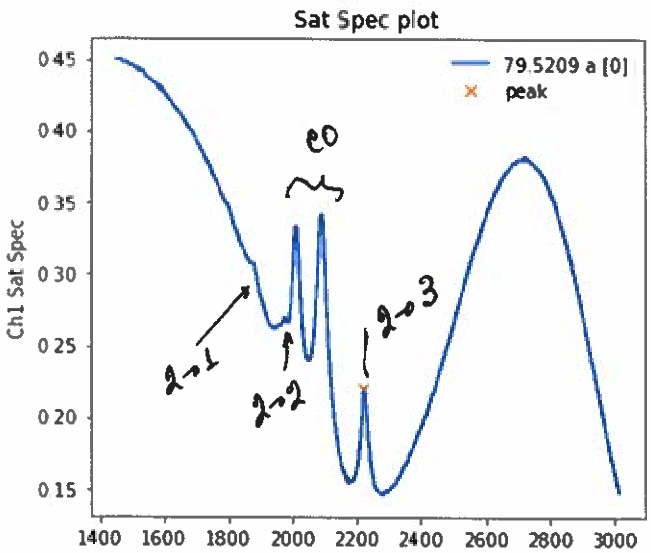

2→1
2→2


$5^2P_{1/2}$
816 MHz
2
1
nm
795 nm
2
= F
6835 MHz
1
^{87}Rb

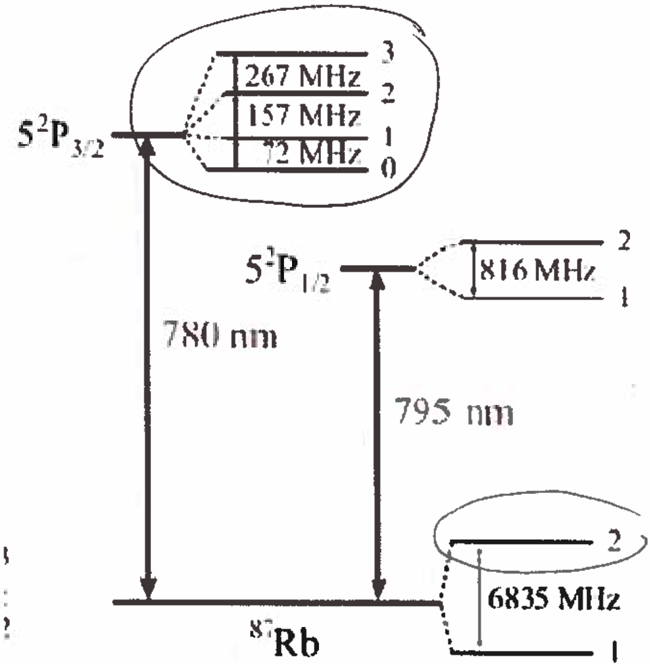

3→2
3→1


$5^2P_{1/2}$
361 MHz
3
2
nm
795 nm
3
= F
3036 MHz
2
^{85}Rb

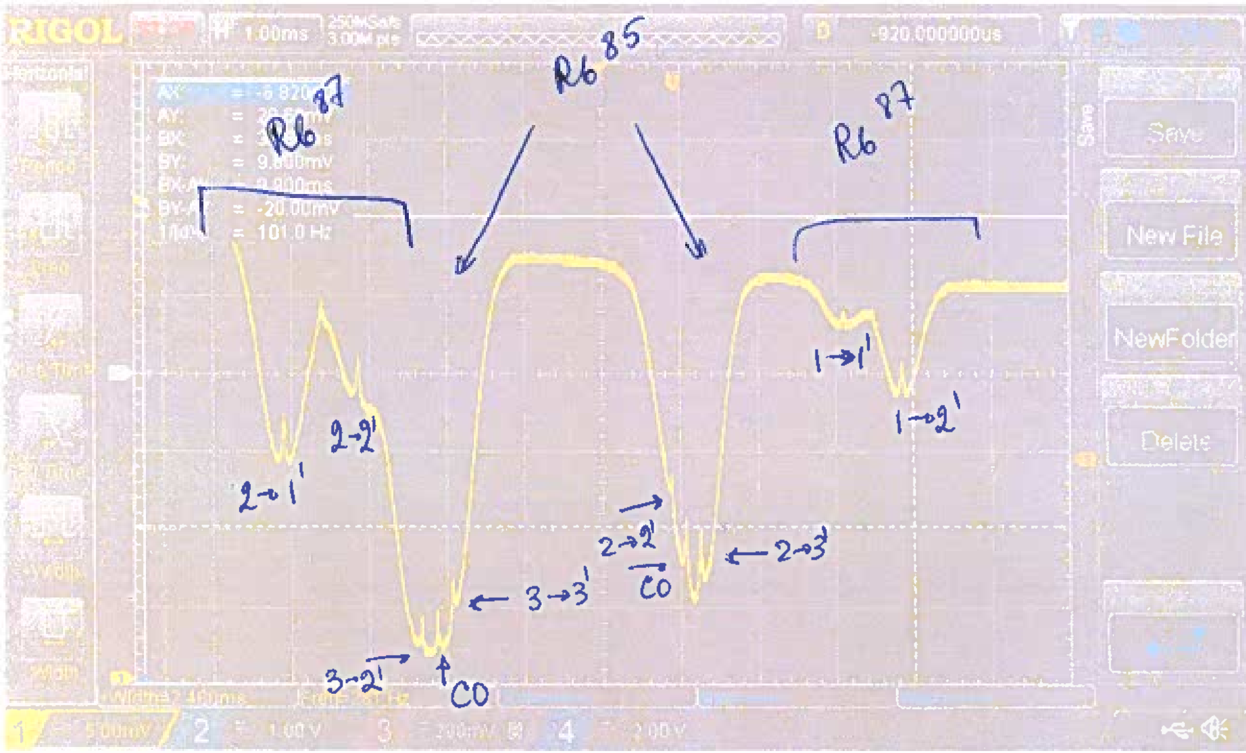
Rb D2 line



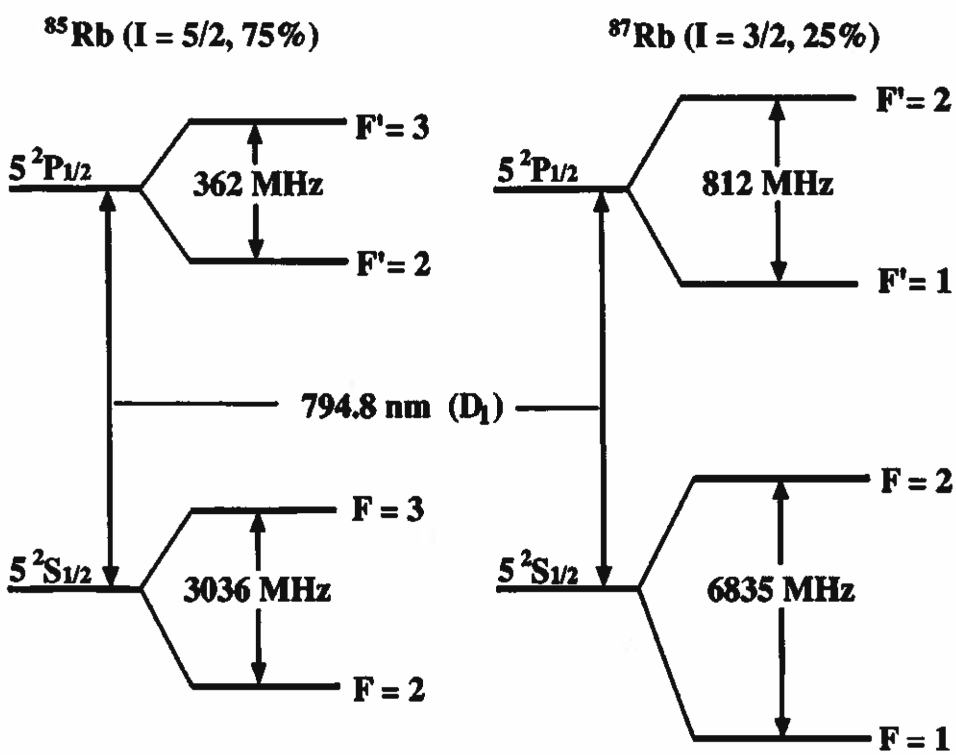
Sat Spec plot
Ch1 Sat Spec
0.45
0.40
0.35
0.30
0.25
0.20
0.15
1400
1600
1800
2000
2200
2400
2600
2800
3000
79.5209 a [0]
peak
CO
2→1
2→2
2→3



3
267 MHz
2
157 MHz
1
72 MHz
0
$5^2P_{3/2}$
2
816 MHz
1
$5^2P_{1/2}$
780 nm
795 nm
2
6835 MHz
1
^{87}Rb



RIGOL
1.00ms
-920.000000us
Save
New File
NewFolder
Delete
Rb 85
Rb 87
Rb 87
2→1'
2→2'
3→2'
CO
3→3'
2→2'
CO
2→3'
1→1'
1→2'



^{85}Rb (I = 5/2, 75%)
^{87}Rb (I = 3/2, 25%)
$5\,^2P_{1/2}$
F' = 3
362 MHz
F' = 2
F' = 2
812 MHz
F' = 1
794.8 nm (D$_1$)
$5\,^2S_{1/2}$
F = 3
3036 MHz
F = 2
F = 2
6835 MHz
F = 1