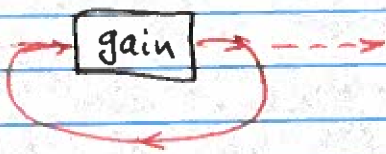
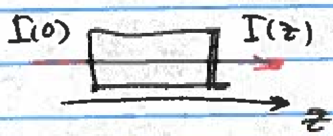
Basics of a laser operation

Necessary components: amplifying medium
optical feedback



We need population inversion to achieve amplification

Gain coefficient $g = \frac{k}{2} \frac{\wp_{ab}^2}{\varepsilon_0 \hbar \gamma_{ab}} N \frac{\mathcal{L}(\Delta)}{1 + I/I_s \cdot \mathcal{L}(\Delta)}$



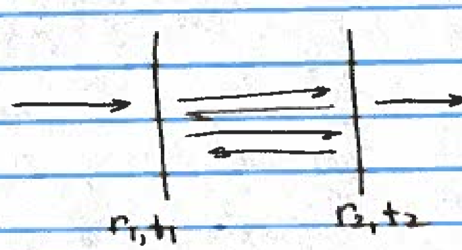
$I(z) = I(0)\, e^{gz}$

$N = \rho_{aa}^{(0)} - \rho_{bb}^{(0)}$
unsaturated population inversion

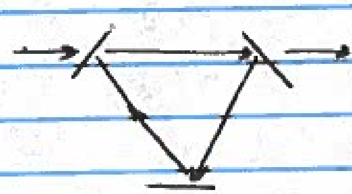
Optical feedback is provided by an optical cavity

In this note I use "N" for unsaturated gain, not $\Delta\rho$

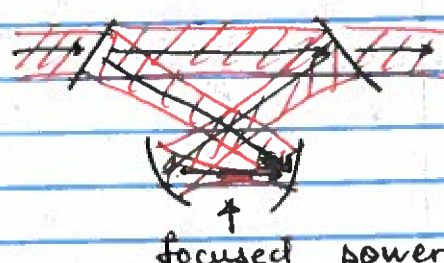
Two-mirror configuration
Fabri-Perot resonator



Ring cavity
(running wave resonator)



Bow-tie cavity
~~(for power~~
(build-up cavity)



Important properties of a resonator:
optical field ~~of~~ can circulate outside repeating itself at each round trip
This usually occurs only at certain frequencies.

Quality factor of a resonator

$$Q = \frac{\text{stored energy (per optical cycle)}}{\text{dissipated power}}$$

If P_{opt} – the length of a round-trip of light inside the cavity and $\ae$ is a fractional loss per round trip, then the dissipated energy per round trip is

$$\frac{d\mathcal{E}}{dt} = -\frac{\ae \mathcal{E}}{T} = -\frac{c\,\ae}{P_{opt}}\,\mathcal{E}$$

where $T = \frac{P_{opt}}{c}$ round-trip time

Photon "lifetime" inside the cavity

$$t_c = \cancel{P_{opt}/c \cdot \ae}\ \frac{P_{opt}}{c \cdot \ae} \qquad (\mathcal{E} = \mathcal{E}(0)\, e^{-t/t_c})$$

$$Q = \frac{\omega \cdot \mathcal{E}}{|d\mathcal{E}/dt|} = \omega \cdot t_c$$

ω – optical frequency

Sources of optical losses
– imperfect reflection of the mirrors (necessary evil)
– absorption and scattering in the mirrors and active medium
– diffraction losses

Resonant frequencies

In order for an optical wave to repeat itself on each round trip

$$p_{opt} / \lambda_m = m \qquad \frac{p_{opt} \cdot \omega_m}{2\pi c} = m$$

Resonant frequencies

$$\omega_m = 2\pi m \frac{c}{p_{opt}} \qquad f_m = m \frac{c}{p_{opt}}$$

equidistant modes

$$f_m - f_{m-1} = c / p_{opt}$$

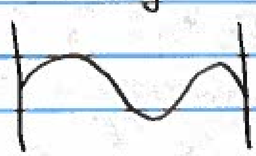
Typically, each mode has associated transverse intensity structure, and often the frequencies of different transverse modes are non-degenerate.

Electric field inside a resonator

$$E(z,t) = \frac{1}{2} \sum_m E_m(t)\, u_m(z)\, e^{-i\omega_m t + \varphi_m} + c.c.$$

Here, $u_m(z)$ is a function describing spatial distribution of individual modes with frequency m

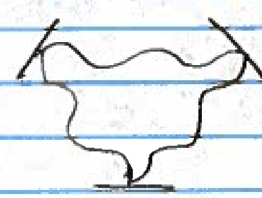
Fabri - Perot cavity



standing wave

$u_m(z) = \sin k_m z$

Ring cavity



running wave

$u_m(z) = e^{ik_m z}$

In general, the spatial mode function depends on the transverse coordinates as well $u_m(z,x,y) = \sin k_{m s_i} z \cdot H_{s_i}(x,y)$ where frequency and phase of each mode are different. However, we are not going to dive into this today. In fact, we can take the spatial variation out. Correspondingly, the induced polarization (assuming the active medium fills the whole cavity)

$$P(z,t) = \sum_m P_m(t)\, e^{-i\omega_m t + \varphi_m}\, u_m(z) + c.c.$$

Let's for simplicity assume a single mode operation: i.e. a single laser frequency circulates.

From the Maxwell's equations

$$\frac{\partial E_m}{\partial t} = \underbrace{-\frac{\omega_m}{2\varepsilon_0} \operatorname{Im} P_m}_{\text{contribution of the gain medium}} \underbrace{- \frac{\omega_m}{2Q_m} E_m}_{\text{cavity losses}}$$

$$\omega_m + \frac{\partial \varphi_m}{\partial t} = c \cdot k_m - \frac{\omega_m}{2\varepsilon_0} \frac{1}{E_m} \operatorname{Re} P_m$$

Substituting $\quad P_m = \varepsilon_0 \chi_m E_m = \varepsilon_0 (\chi_m' + i\chi_m'') E_m$

$$\frac{\partial}{\partial t} E_m = -\frac{\omega_m}{2Q_m} E_m - \frac{\omega_m}{2} \chi_m'' E_m$$

$$\frac{\partial \varphi_m}{\partial t} + \omega_m = k_m c - \frac{\omega_m}{2} \chi_m'$$

Rewriting for the stored energy $\mathcal{E}_m$

$$\frac{\partial \mathcal{E}_m}{\partial t} = \underbrace{-\frac{\omega_m}{Q}\mathcal{E}_m}_{\text{losses}} \underbrace{-\ \omega_m \chi_m'' \mathcal{E}_m}_{\text{gain}} \quad (\text{for } \chi_m'' < 0)$$

Steady-state operation

$$\frac{\partial \varphi}{\partial t} = 0 \qquad \omega_m = k_m c - \frac{\omega_m}{2}\chi_m'$$

using $\chi_m' = -\frac{\Delta}{\Delta^2 + \gamma_{ab}^2}\frac{p_{ab}^2}{\varepsilon_0 \hbar}\frac{N}{1 + I/I_s L(\Delta)} =$

$$= -(\omega_m - \omega_{ab})\frac{p_{ab}^2}{\varepsilon_0 \hbar \gamma_{ab}^2}\frac{N L(\Delta)}{1 + I/I_s L(\Delta)} \propto -(\omega_m - \omega_{ab})$$

$$\omega_m = k_m c + (\omega_m - \omega_{ab}) \cdot \left(\begin{array}{c}\text{positive} \\ \text{function of } \Delta^2\end{array}\right)$$

The generated frequency is "pulled" toward the atomic resonance from the "empty cavity" ~~at~~ frequency

$$\omega_m^{(0)} = k_m \cdot c$$

For calculating the laser intensity we will consider a running-wave cavity, so that there is no spatial variation of intensity

Modal gain $\quad g_m = \frac{\omega_m p_{ab}^2}{2\varepsilon_0 \hbar \gamma_{ab}} N \cdot L(\Delta_m) \cdot \frac{1}{1 + I/I_s L}$

$$\Delta_m = \omega_m - \omega_{ab}$$

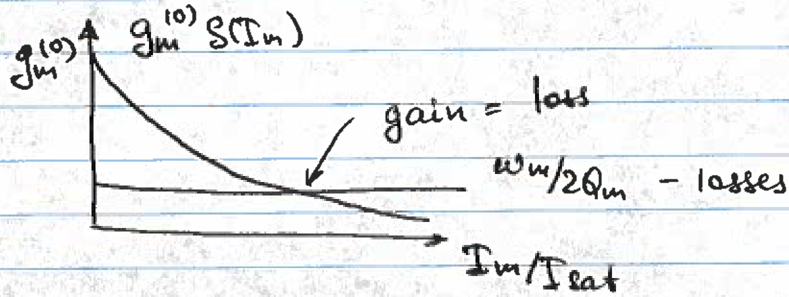
If you ever need it, the expressions for a standing-wave cavity are known, but we won't discuss them here

Equation for the ~~m~~ mode intensity

$$\frac{\partial I_m}{\partial t} = 2 I_m \Big(\underbrace{g_m^{(0)}\, S(I_m)}_{\text{gain}} - \underbrace{\frac{\omega_m}{2Q_m}}_{\text{loss}} \Big)$$

Saturation $\quad S(I_m) = \dfrac{1}{1 + I_m L(\Omega_m)/I_m^{(sat)}} \qquad \left[g_m = g_m^{(0)} S(I_m) \right]$

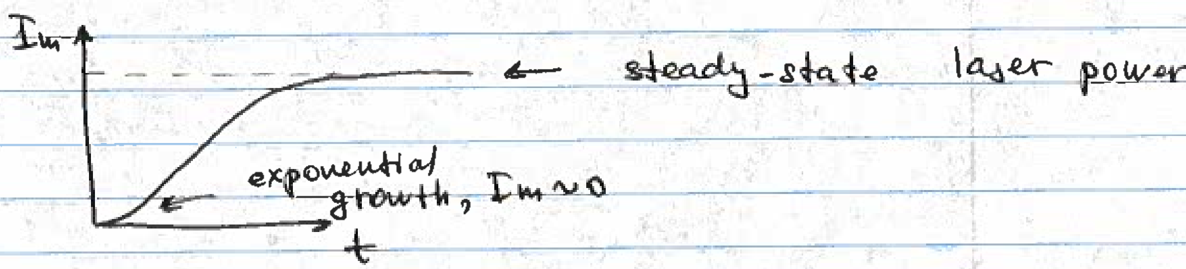
$$S(I_m) \longrightarrow \begin{array}{l} 1, \text{ if } I_m \ll I_m^{(sat)} \\ 0, \text{ if } I_m \gg I_m^{(sat)} \end{array}$$



Small intracavity intensity $\longrightarrow$ large gain
(field strengthens)

when $\quad g_m^{(0)}\, S(I_m) = \omega_m/2Q_m \qquad \dfrac{\partial I_m}{\partial t} = 0$

steady-state solution



Threshold behavior:

unsaturated gain = losses

(minimum requirement for the laser operation)

$$g_m^{(0)} = \frac{\omega_m \wp_{ab}^2}{2\varepsilon_0 \hbar \gamma_{ab}} N_T = \frac{\omega_m}{2Q_m} \quad (\text{assume } \Delta_m = 0)$$

N_T – minimum required population inversion

$$N_T = \frac{\varepsilon_0 \hbar \gamma_{ab}}{\wp_{ab}^2 Q_m}$$

The higher the Q_m the lower is the threshold inversion

~~To~~ Using the notation for N_T

$$g_m^{(0)} = \frac{\omega_m}{2Q_m} \frac{N}{N_T} L(\Delta_m)$$

Above-threshold operation:

$$g_m^{(0)} S(I_m) = \frac{\omega}{2Q_m} \Rightarrow S(I_m) = \frac{1}{g_m^{(0)}} \frac{\omega}{2Q} = \frac{N_T}{N L(\Delta_m)}$$

$$S(I_m) = \frac{1}{1 + I_m / I_m^{(sat)} L(\Delta m)}$$

$$I_m = I_m^{(sat)} \frac{N/N_T \, L(\Delta m) - 1}{L(\Delta m)}$$