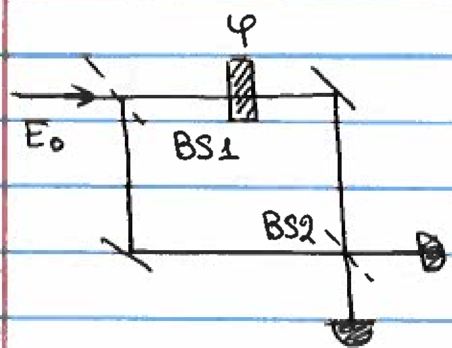
Interferometer – one of the best measurement devices



Objective – to measure the phase shift φ

Input: $E = E_0 \underbrace{e^{ikz - i\omega t}}_{\text{common for all fields, don't write}}$

Input:	E_0	
After BS1:	$\frac{1}{\sqrt{2}} E_0$	$\frac{i}{\sqrt{2}} E_0$
After φ:	$\frac{1}{\sqrt{2}} E_0 e^{i\varphi}$	$\frac{i}{\sqrt{2}} E_0$
After BS2	$\frac{i}{2} E_0 (1+e^{i\varphi})$	$\frac{1}{2} E_0 (1-e^{i\varphi})$

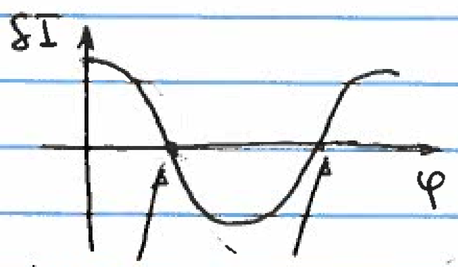
Intensities $\quad I_{1,2} \propto |E_{1,2}|^2 = \frac{|E_0|^2}{4} \left| e^{i\varphi/2} \pm e^{-i\varphi/2} \right|^2$

$$I_1 = I_0 \cos^2 \varphi/2$$

$$I_2 = I_0 \sin^2 \varphi/2$$

Differential intensity $\delta I = I_1 - I_2 = I_0 (\cos^2\varphi/2 - \sin^2\varphi/2)$

$$\delta I = I_0 \cos\varphi$$



best positions to measure $\delta\varphi$ precisely

At what value of φ we are most sensitive to small variations of φ?

$\varphi \approx \pi/2 \qquad \varphi = \frac{\pi}{2} + \delta\varphi \to \sin\delta\varphi \approx \delta\varphi$

Quantum interferometer

Beam-splitter transformation: $\hat{a}_2 = \frac{1}{\sqrt{2}}(\hat{a}_0 + i\hat{a}_1)$

$\cancel{\phi}$ $U_{BS} = \exp\{i\frac{\pi}{4}(\hat{a}_0^+\hat{a}_1 + \hat{a}_0\hat{a}_1^+)$ $\quad \hat{a}_3 = \frac{1}{\sqrt{2}}(\hat{a}_1 + i\hat{a}_0)$

Now we need a phase shifter operator

$$U_\varphi = e^{i\hat{a}^+\hat{a}\varphi}$$

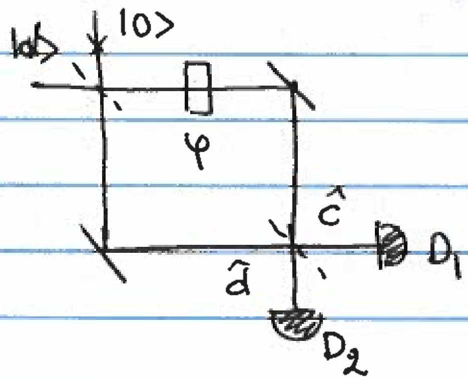
Coherent state: $U_\varphi|\alpha\rangle = e^{i\hat{a}^+\hat{a}\varphi}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}e^{-|\alpha|^2/2}|n\rangle =$

$$= \sum_{n=0}^{\infty} e^{in\varphi}\frac{\alpha^n}{\sqrt{n!}}e^{-|\alpha|^2/2}|n\rangle = \sum_{n=0}^{\infty}\frac{(\alpha e^{i\varphi})^n}{\sqrt{n!}}e^{-|\alpha|^2/2}|n\rangle$$

$= |\alpha e^{i\varphi}\rangle$ The phase shifter operator changes the phase of coherent state by φ, as expected.

(But note: $U_\varphi|n\rangle = e^{in\varphi}|n\rangle$! We will explore that in a moment)

Let's analize the max sensitivity of phase measurements in a Mach-Zender interferometer



$i_{D_1} \propto \langle \hat{c}^+ \hat{c} \rangle$

$i_{D_2} \propto \langle \hat{d}^+ \hat{d} \rangle$

Photocurrent difference operator: $\hat{O} = \hat{c}^+\hat{c} - \hat{d}^+\hat{d}$

A coherent state as an input 1, coherent vacuum — as input 2.

Input: $|0\rangle|\alpha\rangle$

After the first BS: $\left|\frac{i\alpha}{\sqrt{2}}\right\rangle\left|\frac{\alpha}{\sqrt{2}}\right\rangle$

After the phase shifter $\left|\frac{i\alpha}{\sqrt{2}}\right\rangle\left|\frac{\alpha}{\sqrt{2}}e^{i\varphi}\right\rangle$

After the second BS:

$$|\psi_f\rangle = \left|\frac{i(e^{i\varphi}+1)\alpha}{2}\right\rangle\left|\frac{(e^{i\varphi}-1)\alpha}{2}\right\rangle$$

Average measurement outcome

$$\langle\psi_f|\hat{O}|\psi_f\rangle = |\alpha|^2\frac{|e^{i\varphi}+1|^2}{4} - |\alpha|^2\frac{|e^{i\varphi}-1|^2}{4} = |\alpha|^2\cos\varphi$$

same as classical!

This is the signal of the measurement

Noise of the measurements comes from the fluctuations

$$\Delta O = \sqrt{\langle\hat{O}^2\rangle - \langle\hat{O}\rangle^2}$$

For a coherent state $\Delta O \propto \Delta(\delta n) = \sqrt{\Delta n_c^2 + \Delta n_d^2}$

For $|\alpha\rangle$ $\quad \Delta n = \langle n\rangle^{1/2} = |\alpha|$

For $|\frac{e^{i\varphi}\pm 1}{2}\alpha\rangle$ $\quad \Delta n_{c,d} = |\alpha| \begin{matrix}\cos\varphi/2 \\ \sin\varphi/2\end{matrix}$

$$\Delta O = \sqrt{|\alpha|^2\cos^2\varphi/2 + |\alpha|^2\sin\varphi/2} = |\alpha|$$

Uncertainty of the phase measurement

$$\Delta\varphi = \frac{\Delta O}{\left|\frac{\partial O}{\partial\varphi}\right|_{max}} = \frac{|\alpha|}{|\alpha|^2\,|\sin\varphi|} = \frac{1}{|\alpha|\,|\sin\varphi|}$$

Best sensitivity (min $\Delta\varphi$) $\longrightarrow$ $\sin\varphi = \pm 1$
just like for the classical state

$$\Delta\varphi_{min} = \frac{1}{|\alpha|} = \frac{1}{\sqrt{\langle n\rangle_\alpha}} \longleftarrow \text{shot noise limit}$$

This is the fundamental limit for any measurements using a coherent input state

However, this is much less sensitive than Heisenberg limit

$$\Delta n \cdot \Delta\varphi \gtrsim 1 \qquad \Delta\varphi_{Heis} \sim \frac{1}{\langle n\rangle}$$

How can we improve the sensitivity?

- send squeezed vacuum to the second input port
- use non-coherent states for the detection.

Single photon interferometer

Input state $|0\rangle|1\rangle$

After the first BS: $\frac{1}{\sqrt{2}}(|0\rangle|1\rangle + i|1\rangle|0\rangle)$

After the phase shifter $\frac{1}{\sqrt{2}}(e^{i\varphi}|0\rangle|1\rangle + i|1\rangle|0\rangle)$

After the second BS:

$$\left[\begin{array}{l} |0\rangle|1\rangle \longrightarrow \frac{1}{\sqrt{2}}(|0\rangle|1\rangle + i|1\rangle|0\rangle) \\ |1\rangle|0\rangle \longrightarrow \frac{1}{\sqrt{2}}(|1\rangle|0\rangle + i|0\rangle|1\rangle) \end{array}\right] \quad \text{each component transforms}$$

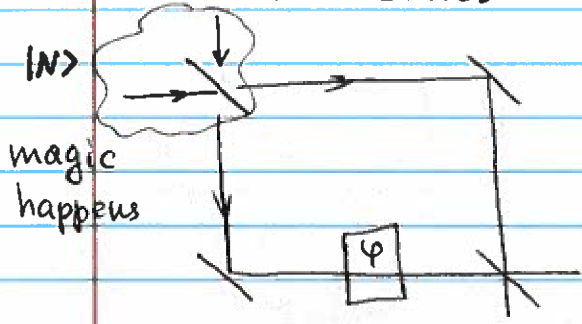
$$\psi_{fin} = \frac{1}{2}\left\{ e^{i\varphi}(|0\rangle|1\rangle + i|1\rangle|0\rangle) + i(|1\rangle|0\rangle + i|0\rangle|1\rangle)\right\} =$$

$$= \frac{1}{2}\left\{ (e^{i\varphi}-1)|0\rangle|1\rangle + i(e^{i\varphi}+1)|1\rangle|0\rangle \right\}$$

Each detector will click with probabilities $P_{1,2} = \frac{1}{4}|1 \pm e^{i\varphi}|^2 = \begin{matrix}\cos^2\varphi/2 \\ \sin^2\varphi/2\end{matrix} = \frac{1}{2}(1 \pm \cos\varphi)$

$\langle \psi_{fin}|\hat{O}|\psi_{fin}\rangle = \cos\varphi$ (similar to coherent case)

NOON states



After the magic beam splitter

$$\frac{1}{\sqrt{2}}\left(|N\rangle_1|0\rangle_2 + |0\rangle_1|N\rangle_2\right)$$

(NOON state)

After the phase-shifter

$$e^{i\hat{n}\varphi}|N\rangle_2 = e^{iN\varphi}|N\rangle_2$$

$$|N\rangle_1|0\rangle_2 + e^{iN\varphi}|0\rangle_1|N\rangle_2$$

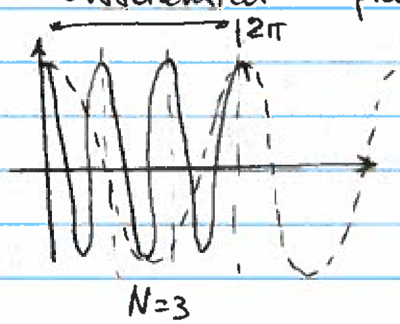
After the second beam splitter

$$\frac{1}{2}\left\{(e^{iN\varphi}-1)|0\rangle_1|N\rangle_2 + i(e^{iN\varphi}+1)|N\rangle_1|0\rangle_2\right\}$$

Probabilities of detection in each channel

$$P_{1,2} = \frac{1}{2}(1 \pm \cos N\varphi)$$

Differential photocount system $\Delta I \propto \cos N\varphi$



Supersensitivity

$$\Delta\varphi \propto \frac{1}{N}$$

in principle can reach the Heisenberg limit

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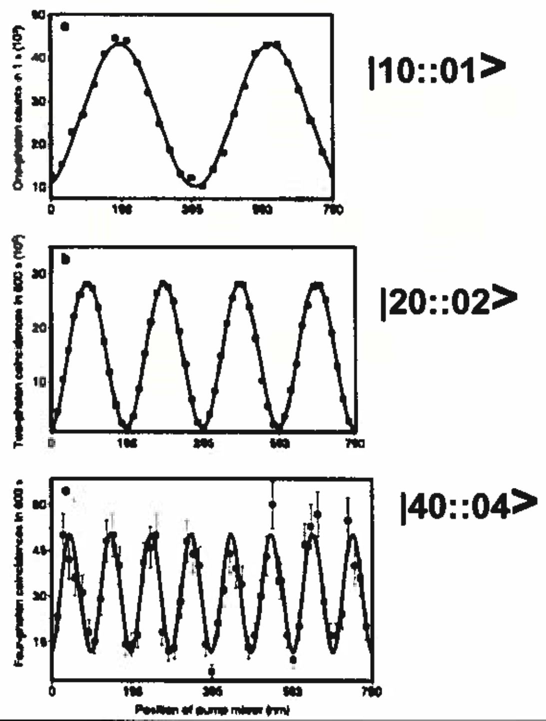
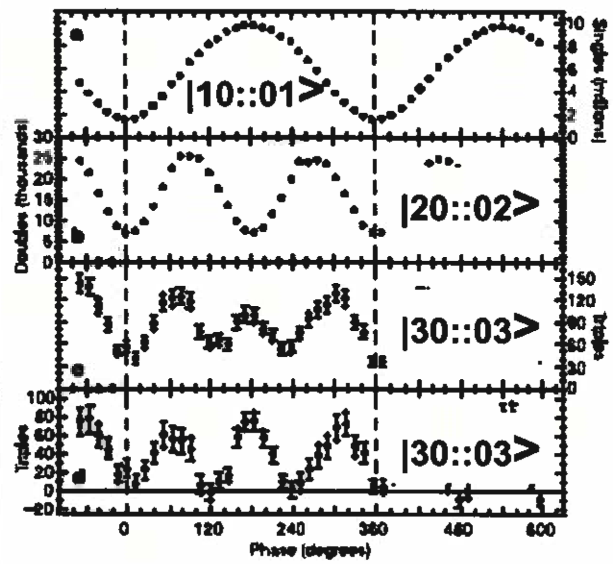
De Broglie wavelength of a non-local four-photon state

Philip Walther[1], Jian-Wei Pan[1,*], Markus Aspelmeyer[1], Rupert Ursin[1], Sara Gasparoni[1] & Anton Zeilinger[1,2]



Super-resolving phase measurements with a multiphoton entangled state

M. W. Mitchell, J. S. Lundeen & A. M. Steinberg



Quantum P00Per Scooper!

H Cable, R Glasser, & JPD, quant-ph/0704.0678.

2-mode squeezing process

X

OPO

$\sum_n p_n |n\rangle|n\rangle$

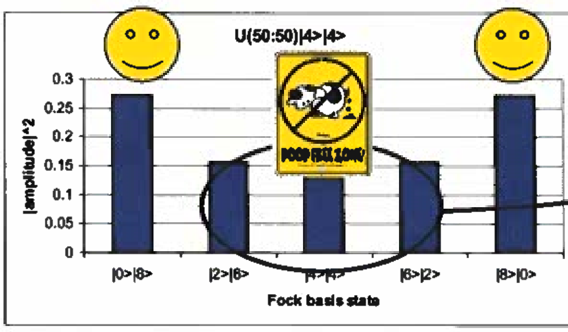
Old Scheme

linear optical processing

$\frac{|N0\rangle + |0N\rangle}{\sqrt{2}}$

beam splitter

New Scheme



How to eliminate the "POOP"?

quant-ph/0608170
G. S. Agarwal, K. W. Chan, R. W. Boyd, H. Cable and JPD