Physics 611, Fall 2014

Problem set #3 (due October 27)

1. A He-Ne laser with wavelength $\lambda=633\text{nm}$ nominally $L=1\text{m}$ long is to be designed with a hemispherical cavity, i.e. one flat mirror and one curved mirror with $R=1\text{m}$. A micrometer screw is to be used to vary the exact cavity length over a small range, so that the cavity length will be $L=R-\Delta L$, where $\Delta L\ll L$. In this way, the spot size $w_2$ at the curved-mirror end can be varied to fill the 5mm radius of an aperture placed there.
   a. Write down the simple expression for $w_2$ as a function of $\Delta L$.
   b. Over what range of $\Delta L$ must the micrometer screw move the curved mirror if $w_2$ is to vary from 5mm to all larger values?
   c. When $w_2 = 5\text{mm}$, what is the value of the beam size $w_1$ at the flat mirror end of the laser?

2. If we consider a higher-order $(n,m)$ Hermit-Gaussian mode in a cavity, consisting of two spherical mirrors with radii $R_1$ and $R_2$ and separated by the distance $L$, the expression for its Guoy phase is $\varphi(z) = (n + m + 1)\tan^{-1}\left(\frac{z}{z_R}\right)$, where $z_R$ is the Rayleigh range, $l$ is axial mode index, and $n$ and $m$ are the transverse mode indices. Using this expression, show that the corresponding frequencies for axial and transverse modes are $f_{lmn} = c/p\left(l + \frac{(n+m+1)}{\pi}\cos^{-1}\sqrt{g_1g_2}\right)$, where $p$ is the longitudinal optical path in the cavity, and $g_i=1-L/R_i$.

3. For the same cavity as in Problem 1 (a hemispherical geometry, $L=R-\Delta L$, where $\Delta L\ll L,R$), assume that the laser is allowed to oscillate in several axial and transverse modes, and that the beat frequency $f_{\text{beat}}$ is the lowest intermode beat frequency that is observed in the laser output. Verify that $w_2^2 \cdot f_{\text{beat}} = c\lambda/\pi^2$, independent of $L$ or $R$, as $\Delta L$ is varied with $\Delta L\ll L$.

4. A collimated Gaussian beam of a fixed spot size $w$ is to be focused to the absolute minimum possible spot size (not necessarily a beam waist) of a work piece, using a single lens located a fixed distance $L$ from the work piece. What should be the exact focal length $f$ of this lens, and what will be the exact spot size of the focused spot?

5. Jackson 8.2
6. Jackson 8.3
7. Jackson 8.6
8. Jackson 8.19