PHYS 404/690 Quantum and Nonlinear Optics

Problem set # 3 *(due January 20)*

Each problem is 10 points. The problems marked with $*$ are required for graduate students only, and are extra credit problems for undergraduates.

P1 Consider a three-level atom in the Λ configuration. The on-resonant interaction Hamiltonian in this case is
$\hat{H}_{int} = -\frac{\hbar}{2}\left(\Omega_1|3\rangle\langle 1| + \Omega_2|3\rangle\langle 2|\right) + c.c.$,
where Ω_1 and Ω_2 are the Rabi frequencies associated with the optical fields driving $3 \to 1$ and $3 \to 2$ transitions, respectively.

Show that the eigenstates of the Hamiltonian are:

$$|\psi_+\rangle = \frac{1}{\sqrt{2}}\left(|3\rangle + \frac{\Omega_1^*}{\Omega}|1\rangle + \frac{\Omega_2^*}{\Omega}|2\rangle\right);$$
$$|\psi_0\rangle = \left(\frac{\Omega_2}{\Omega}|1\rangle - \frac{\Omega_1}{\Omega}|2\rangle\right);$$
$$|\psi_-\rangle = \frac{1}{\sqrt{2}}\left(|3\rangle - \frac{\Omega_1^*}{\Omega}|1\rangle - \frac{\Omega_2^*}{\Omega}|2\rangle\right);$$

with $\Omega = \sqrt{|\Omega_1|^2 + |\Omega_2|^2}$. Find the corresponding eigenvalues.

P2 Using the density matrix elements equations we wrote in class for a Λ system as an example, write down the similar equations for a V system, in which two optical fields with Rabi frequencies Ω_1 and Ω_2 connect two excited states $|1\rangle$ and $|2\rangle$ with the common ground state $|3\rangle$. Assume that the two excited states have same population and optical coherence decay rates into the ground state γ_p and γ_c, correspondingly. Also assume that there is no population exchange between the two excited states, but there is a non-zero collisional decoherence with the rate γ_{12}.

P3 Let's have a closer look at the optical pumping effect. We assume a single optical field with Rabi frequency Ω acting on a three-level atom, between the states $|2\rangle$ and $|3\rangle$. If there is a population exchange between the two ground states at the rate γ_0, the non-zero elements of the density matrix evolve as:

$$\dot{\rho}_{11} = \gamma\rho_{33} - \gamma_0(\rho_{11} - \rho_{22});$$
$$\dot{\rho}_{22} = \gamma\rho_{33} + \gamma_0(\rho_{11} - \rho_{22}) - i\Omega\rho_{23}/2 + i\Omega^*\rho_{32}/2;$$
$$\dot{\rho}_{32} = -(\gamma - i\Delta)\rho_{32} + i\Omega(\rho_{22} - \rho_{33})/2;$$

where γ is the population and decoherence decay rate of the state $|3\rangle$ into each of the ground states, and Δ is the optical detuning.

Calculate the steady-state populations of all three levels.

P4* We can obtain insight into the physical origin of the differences between the two absorption resonances in a far-detuned three-level system by examining it in the dressed state picture that you have studied in HW2. Show the strong field Ω_2 "dresses" the two-level transition $2 \to 3$ bare three-level system, so that the eigenstates probed by Ω_1 contain a mixture of states $|2\rangle$ and $|3\rangle$:

$$|+\rangle \simeq |3\rangle + \frac{\Omega_2^*}{2\Delta}|2\rangle;$$
$$|-\rangle \simeq |2\rangle - \frac{\Omega_2}{2\Delta}|3\rangle;$$

where we have only kept terms to lowest order in Ω_2/Δ because of the large detuning. Using this result, explain the expected probe absorption features.