PHYS 404/690 Quantum and Nonlinear Optics

Problem set # 2 *(due January 6)*

Each problem is 10 points. The problems marked with $*$ are required for graduate students only, and are extra credit problems for undergraduates.

P1 Draw the level diagram for and white the expression for the interaction Hamiltonian to the three-level system with the equations of motion:
$\dot{c}_1 = \frac{i}{2}\mathcal{R}c_2$
$\dot{c}_2 = \frac{i}{2}\mathcal{R}(c_1 + c_3)$
$\dot{c}_3 = \frac{i}{2}\mathcal{R}c_2$

P2 In class we derived expression for Rabi oscillations assuming no decay of the atomic states. While usually it is impossible to properly include spontaneous emission is a wave-function description of the semi-classical light-atom interactions, in the case of the two-level atom the finite lifetime of the atomic levels can be described by adding phenomenological decay terms to the probability amplitude equations:

$$\begin{aligned} \dot{c}_e &= -\frac{\gamma}{2}c_e - i\frac{\Omega}{2}c_g \\ \dot{c}_g &= -\frac{\gamma}{2}c_g - i\frac{\Omega}{2}c_e \end{aligned} \tag{1}$$

where γ is the decay rate. For an atom initially in the state $|e\rangle$, show that the inversion (differences in populations between the excited and the ground states) is $P_e - P_g = e^{-\gamma t}cos(\Omega t)$.

P3 Using explicit expressions for the wave functions for the hydrogen atom, explain which transitions are possible between $1S$ state and various m-sublevels of $2P$ states, in case of x, y and $\sigma_\pm$ polarized electromagnetic field. Assume that the light propagates along z axis, and it is also the quantization direction for the atoms.

P4* A useful alternative basis for the two-level system cosists of *dressed states*, which are the eigenvectors of the Hamiltonian $\hat{H} = \hat{H}_0 + \hat{H}_I$. Specifically, show that the eigenvectors satisfy the eigenvalue equations
$\hat{H}\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} -\delta & \Omega_0 \\ \Omega_0 & \delta \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix} = \lambda\begin{bmatrix} u \\ v \end{bmatrix}$
are given by
$\begin{bmatrix} u_2 \\ v_2 \end{bmatrix} = \frac{1}{\sqrt{(\Omega-\delta)^2+\Omega_0^2}}\begin{bmatrix} \Omega - \delta \\ \Omega_0 \end{bmatrix} = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$
for the eigenvalue $\lambda = \Omega$ and
$\begin{bmatrix} u_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}$
for $\lambda = -\Omega$. Here Ω_0 is the Rabi frequency of the transition, $\delta = \omega_{ab} - \omega$ is the difference between the frequencies of atomic transition and the electromagnetic field, and $\Omega = \sqrt{|\Omega_0|^2 + \delta^2}$ is the generalized Rabi frequency.