PHYS 404/690 Quantum and Nonlinear Optics

Problem set # 10 *(due April 24)*

Each problem is 10 points. The problems marked with $*$ are required for graduate students only, and are extra credit problems for undergraduates.

P1 Use the Bose commutation relationship between the incoming and the outgoing modes in order to show that the amplifier matrix A obeys the relations:
$|A_{11}|^2 - |A_{12}|^2 = |A_{22}|^2 - |A_{21}|^2 = 1$, $A_{11}^* A_{21} - A_{12}^* A_{22} = 0$.
Verify that $\hat{a}_1^\dagger \hat{a}_1 - \hat{a}_2^\dagger \hat{a}_2$ is conserved.

P2 The output of a parametric optical amplifier, seeded with a coherent vacuum signal, is described by the following wave function:
$|\psi\rangle = \frac{1}{\cosh\xi} \sum_{n=0}^{\infty} (\tanh\xi)^n |n, n\rangle$,
where ξ is the parameter describing the nonlinearity of the amplifier medium. Show that in case of the weak interaction $\xi \ll 1$ such state predominantly lead to the generation of single pairs, namely that:
$|\psi\rangle \simeq |0, 0\rangle + \xi|1, 1\rangle$.
How would you use such system to implement a source of single photons?

P3 Using the general form of the OPA output wavefunction from the previous problem, show that each of two output fields, if tested individually, is in a thermal state.

P4 Alternative way of testing hidden variable theories involves three-particle correlated states, known as Greenberger-Horne-Zeilinger (GHZ) equality.
Consider a three-particle state $|\Psi_{GHZ}\rangle = (|\uparrow_1\uparrow_2\uparrow_3\rangle - |\downarrow_1\downarrow_2\downarrow_3\rangle)/\sqrt{2}$. What are the expected measurement outcomes of the following operators: $\hat{\sigma}_x^{(1)}\hat{\sigma}_y^{(2)}\hat{\sigma}_y^{(3)}$, $\hat{\sigma}_y^{(1)}\hat{\sigma}_x^{(2)}\hat{\sigma}_y^{(3)}$, $\hat{\sigma}_y^{(1)}\hat{\sigma}_y^{(2)}\hat{\sigma}_x^{(3)}$ and $\hat{\sigma}_x^{(1)}\hat{\sigma}_x^{(2)}\hat{\sigma}_x^{(3)}$ for a particle in the $|\Psi_{GHZ}\rangle$ state?

P5* Let's attempt to recreate the GHZ equality. A hidden variable theory assumes that each possible measurement outcome is "inscribed" within a particle state at the moment of entanglement creation, and is not changed afterwards. So let's assign the corresponding possible outcomes $m_x^{(i)}$ and $m_y^{(i)}$ to each measurement $\hat{\sigma}_x^{(i)}$ or $\hat{\sigma}_y^{(i)}$, such that $m = \pm 1$. Show that the outcome of the direct $\hat{\sigma}_x^{(1)}\hat{\sigma}_x^{(2)}\hat{\sigma}_x^{(3)}$ measurement contradicts the outcome deduced from the separate measurements of $\hat{\sigma}_x^{(1)}\hat{\sigma}_y^{(2)}\hat{\sigma}_y^{(3)}$, $\hat{\sigma}_y^{(1)}\hat{\sigma}_x^{(2)}\hat{\sigma}_y^{(3)}$ and $\hat{\sigma}_y^{(1)}\hat{\sigma}_y^{(2)}\hat{\sigma}_x^{(3)}$ under the hidden variable approximation.