Interaction of an atom with electro magnetic field

Electromagnetic field is oscillating electric and magnetic fields

$$\vec{E} = \vec{E}_0 \cos(\omega t - kz)$$
$$\vec{B} = \vec{B}_0 \cos(\omega t - kz)$$

The strongest interaction of an e-m field with an electron — electro-dipole interaction:

$$U_{el} = -\vec{d}\vec{E} = +e\vec{r}\cdot\vec{E}_0 \cos(\omega t - k\cancel{x})$$

Important to note: the size of an atom (i.e. change of r) $\sim a_0 \sim 1\mathring{A}$, and for most em spectrum $1/k \sim \lambda \sim 100\,nm$ or more. Thus, we can neglect any spatial variations of e-m field across an atom: $\vec{E} = \vec{E}_0 \cos\omega t$

$$U_{el} = e\,\vec{r}\cdot\vec{E}_0 \cos\omega t \quad \text{— periodic perturbation}$$

Quantum version

$$\hat{H}' = e\,\hat{\vec{r}}\cdot\vec{E}_0 \cos\omega t$$

$\vec{E}_0$ is a constant vector in y-z plane (since we assume that the e-m field propagates in z direction).

If we consider the first-order perturbation of this e-m field close to a transition b/w two particular levels a and b

The direction of $\vec{\mathcal{E}}_0$ determine the polarization of e-m field. $\vec{\mathcal{E}}_0 = \mathcal{E}_0 \vec{e}_0 \quad |\vec{e}_0| = 1$

E_b, ψ_b

E_a, ψ_a

$\hat{H}_0$

$\omega_0 = \dfrac{E_b - E_a}{\hbar}$

resonance freq.

$\langle \psi_a | \hat{H}' | \psi_b \rangle = e \mathcal{E}_0 \langle \psi_a | \vec{r} \cdot \vec{e}_0 | \psi_b \rangle \cos\omega t$

$\langle \psi_a | \hat{H}' | \psi_a \rangle = \langle \psi_b | \hat{H}' | \psi_b \rangle = 0$

due to parity ($\vec{r}$ is odd)

[that justifies our previous assumption about $\hat{H}'$ being off-diagonal]

$e \langle \psi_a | \vec{r} \cdot \vec{e}_0 | \psi_b \rangle = \wp_{ab}$ ($\wp$ is called "wiggly p", seriously)

Typically $\wp_{ab}$ is called a transition matrix element or transition strength, and it is detremined by the spatial properties of the involved energy levels. [selection rules!]

$$\langle \psi_a | \hat{H}' | \psi_b \rangle = \underbrace{\wp_{ab} \mathcal{E}_0}_{V_{ab}} \cos\omega t$$

Thus, we can easily relate the results from last lecture to the e-m wave excitation:

- off-resonant case, system is initially in state a

$$P_b = \left(\frac{\wp_{ab} \mathcal{E}_0}{\hbar} \right)^2 \frac{\sin^2 \frac{\omega - \omega_0}{2} t}{(\omega - \omega_0)^2}$$

- resonant case $\omega = \omega_0$

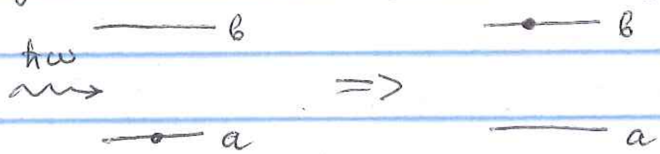
$$P_b = \cancel{\cos} \sin^2 \left[\left(\frac{\wp_{ab} \mathcal{E}_0}{2\hbar} \right) \cdot t \right] = \sin^2 \Omega t \qquad \Omega = \frac{\wp_{ab} \mathcal{E}_0}{2\hbar} \text{ Rabi freq.}$$

In general,

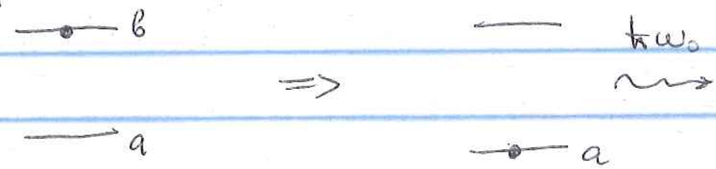
$$P_b = \left(\frac{\Omega}{\sqrt{\Omega^2 + (\omega - \omega_0)^2}} \right)^2 \cancel{\cos} \sin^2 \left[\frac{1}{2} \sqrt{\Omega^2 + (\omega - \omega_0)^2}\, t \right]$$

Absorption and emission of light

It is the easiest to visualize the processes of absorption and emission if we consider light to be a stream of n particles each carrying energy $(n+\frac{1}{2})\hbar\omega$. Absorption of one photon gives an electron enough energy to get to a higher energy level



similarly, when an electron jumps from higher energy level to a lower energy level, an extra photon can be emitted



$\hbar\omega_0$ or $\hbar\omega$, ... or some~~thing~~ similar frequency!

How to account for such energy loss / addition for a classical e-m wave? Maxwell equations!

$$\nabla\times\vec{H} = \cancel{\vec{J}}^{=0} + \frac{\partial\vec{D}}{\partial t}$$

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\nabla\cdot\vec{D} = \rho = 0$$

$$\nabla\cdot\vec{B} = 0$$

$$\vec{D} = \varepsilon_0\vec{E} + \vec{P}$$

$$\vec{B} = \mu_0\vec{H}$$

atomic polarization

Using these we can obtain a wave equation for $\vec{E}$ (or $\vec{H}$)

atomic susceptibilyty

Since $\nabla\times(\nabla\times\vec{E}) = \nabla\cdot\nabla\vec{E} - \nabla^2\vec{E}$

usually $\vec{P} = \varepsilon_0 \chi \vec{E}$

then $\frac{1}{2}\operatorname{Im}(\chi)\cdot\frac{2\pi}{\lambda}$ – absorption coefficient

$$\nabla^2\vec{\mathcal{E}} - \frac{1}{c^2}\frac{\partial^2\mathcal{E}}{\partial t^2} = \mu_0\frac{\partial^2\vec{P}}{\partial t^2}$$

If $\vec{P}=0$ (no atoms) — e-m wave in vacuum

Atomic polarization $\vec{P} = \frac{\text{dipole moment}}{\text{volume}} = N\langle\vec{d}\rangle$

average dipole moment of an atom

$\langle\vec{d}\rangle = \langle\psi|-e\vec{r}|\psi\rangle$ where ψ is the wave function describing the state of an atom

$\psi(t) = c_a(t)\psi_a + c_b(t)\psi_b$

$\langle\vec{d}\rangle = \langle c_a\psi_a + c_b\psi_b|-e\vec{r}|c_a\psi_a + c_b\psi_b\rangle = |c_a|^2\langle\psi_a|-e\vec{r}|\psi_a\rangle +$
$+ |c_b|^2\langle\psi_b|-e\vec{r}|\psi_b\rangle + c_a c_b^*\langle\psi_b|-e\vec{r}|\psi_a\rangle + c_a^* c_b\langle\psi_a|-e\vec{r}|\psi_b\rangle$

$\langle\psi_a|-e\vec{r}|\psi_a\rangle = \langle\psi_b|-e\vec{r}|\psi_b\rangle = 0$ (due to symmetry)

Typically $\vec{P}\uparrow\uparrow\vec{E}$ (and $\vec{d}\uparrow\uparrow\mathcal{E}$) so $\vec{d} = d\cdot\vec{e}_0$

$\langle d\rangle = c_a c_b^*\langle\psi_b|-e\vec{r}\cdot\vec{e}_0|\psi_a\rangle + c_a^* c_b\langle\psi_a|-e\vec{r}\vec{e}_0|\psi_b\rangle =$
$= c_a c_b^*\wp_{ba} + c_a^* c_b\wp_{ab} = 2c_a c_b\wp_{ab}$

if everything is real

Thus we can calculate the effect of atomic interaction on the amplitude of the e-m field

$$\nabla^2\mathcal{E} - \frac{1}{c^2}\frac{\partial^2\mathcal{E}}{\partial t^2} = 2\mu_0 N\wp_{ab}\, c_a c_b$$

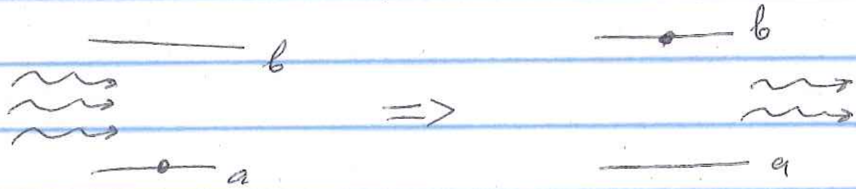
For the resonant case $c_a c_b = \cos\Omega t\,\sin\Omega t = \frac{1}{2}\sin 2\Omega t$

$$\nabla^2\mathcal{E} - \frac{1}{c}\frac{\partial^2\mathcal{E}}{\partial t^2} = \mu_0 N\wp_{ab}\,\underline{\sin 2\Omega t}$$

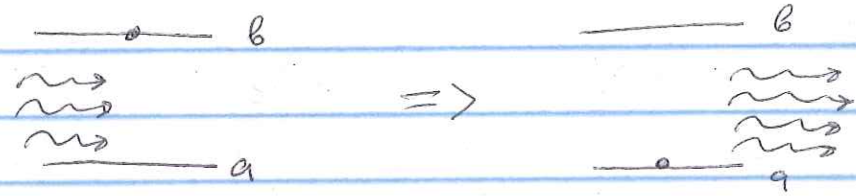
So as a function of time the amplitude is either reduced or amplified by the atomic responce, but if we average the atomic polarization over time longer than $1/\Omega$, the total effect is zero. On average no energy is lost or gained!

Remember – our calculations only include what we added in. So far we only allowed one particular mode of e-m field. Thus, we only allow ~~light~~ to absorb and emit light that is identical to our input em field. Thus, the absorption and emission we consider are stimulated by the presence of the field

Stimulated absorption

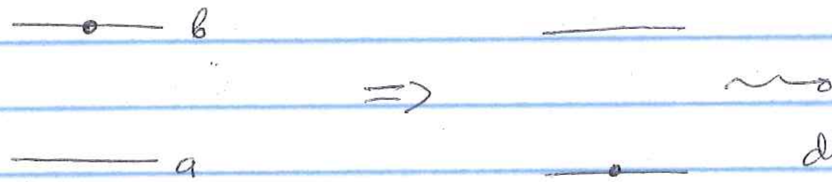


Stimulated emission



emitted photon is indistinguishable from the original field

Spontaneous emission



a photon emitted in a random direction and random polarization

Energy of the emitted photon must obey energy conservation ($\approx \hbar\omega_0$) within uncertainty relationship $\Delta E \cdot \Delta t \gtrsim \hbar$, where $\Delta E = \hbar \Delta\omega$ – photon energy uncertainty, and Δt – finite lifetime of the excited state