Inelastic scattering

collisions (scattering)

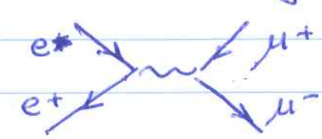
Elastic scattering
a particle A bounces off another particle B w/o changing internal structure of either one
(that's what we discussed so far)

Inelastic collisions

Internal structure of A and/or B changes

$e^- + H \rightarrow e^- + H^*$

Rearrangement collision



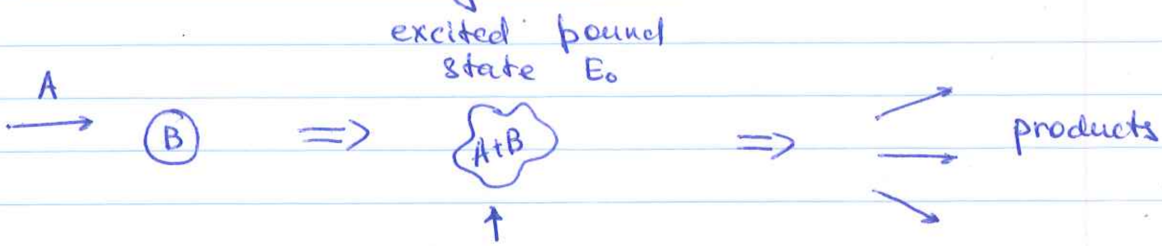
1D
~~Elastic~~ collisions

Re^{-ikx} $\longleftarrow$

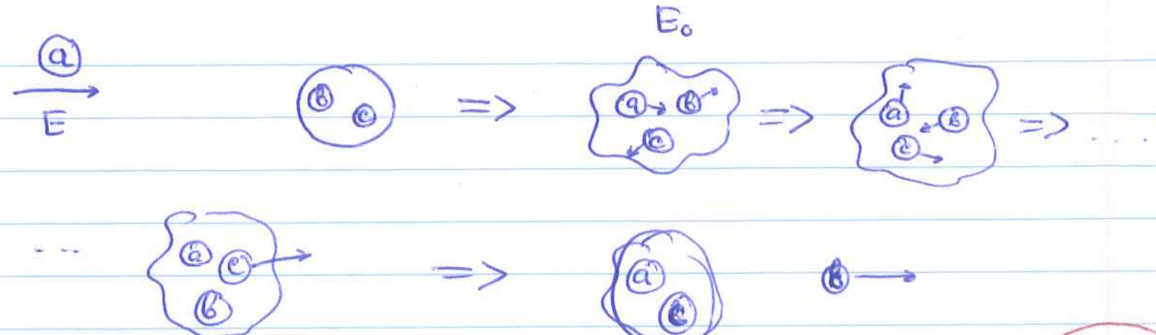
$\longrightarrow$ e^{ikx}

$\lvert R\rvert^2 = 1$	$\lvert R\rvert^2 < 1$
elastic	inelastic

Resonant scattering



the longer this intermediate structure lives, the more time the various components have to redistribute the momentum and have a particular product to obtain enough energy to escape

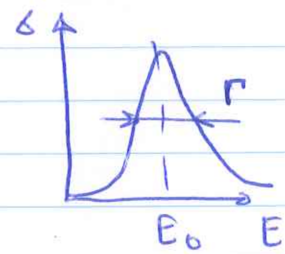


$$f^{(b)}_{ab} = \underbrace{\frac{1}{2ik_a}\left(e^{2i\delta_a}-1\right)\delta_{ab}}_{\substack{\text{elasctic scattering}\\ a \,\longrightarrow\, a}} - \underbrace{\frac{1}{2\sqrt{k_a k_b}}\, e^{i(\delta_a+\delta_b)}\, \frac{\Gamma M_{ab}}{E-E_0+i\Gamma/2}}_{\text{inelastic scattering}}$$

resonance

Inelastic scattering cross-section

$$\sigma(E) \propto \frac{\Gamma^2}{(E-E_0)^2+\Gamma^2/4}$$



$\Gamma = \hbar/\tau$ $\quad$ τ = lifetime of the intermediate state

$\Delta E \cdot \tau \sim \hbar$

uncertainty principle

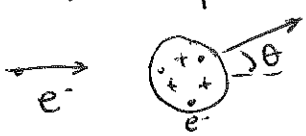
The cross-section is none-zero at any energy E, but it has a peak at energies close to the rest-mass energy E_0 of the intermediate particle. Longer lived intermediate particles have ~~same~~ smaller $\Gamma = 1/\tau$ and hence sharper peaks (aka resonance)

Thus, from the observation of resonances in scattering cross-section we are able to deduce the mass and lifetime of unstable particles!

In search of a nuclear structure

Atomic structure

plum pudding



$$\frac{d\sigma}{d\Omega} \sim e^{-\theta^2/\theta_m^2}$$

nucleus

$$\frac{d\sigma}{d\Omega} = \left(\frac{ZZ'e^2}{16\pi\varepsilon_0 E}\right)^2 \frac{1}{\sin^4\frac{\theta}{2}}$$

observation of non-zero high-angle scattering convinced Rutherford (and the rest of the world) that there is a small positively charged nucleus in the center of an atom.

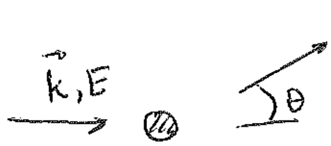
Next step – what is inside a nucleus
→ protons and neutrons (tightly packed)
(established using elastic electron scattering)

Next step – are proton and neutron elementary, point-like particles?

A scattering on a point-like particle (just like Rutherford scattering)

$$\frac{d\sigma}{d\Omega} \sim \frac{e^4 E^2}{q^4}$$

where $\vec{q} = \vec{k}' - \vec{k}$ is the change in scatterer's momentum



$$q^2 = 4EE'\sin^2\theta/2$$

Often Q^2 is used for the momentum transfer, although then it is usually relativistic 4-momentum

In general, for any scatterers' distribution

$$\frac{d\sigma}{d\Omega} = \frac{4e^2 E'^2}{q^4}\left[\underbrace{W_2 \cdot \cos^2\frac{\theta}{2}}_{\text{small-angle scattering}} + \underbrace{2W_1 \cdot \sin^2\frac{\theta}{2}}_{\text{large-angle scattering}}\right]$$

W_1 and W_2 are the structure functions of a proton (or neutron) that contain all available information about their structure

In general $W_{1,2}$ ~~de~~ may depend on q^2 and $\nu = E - E'$ transfered momentum and energy lost.

Once the energy on the incoming particles become high enough, the descrepancies b/w experiment and theoretical predictions for solid uniform sphere (Mott's model) start to diverge.

Bjorken prediction — if a nucleus contains its own ~~solid~~ point-like center or several point-like scatterers, W_2 and W_1 must depend only on ν/q^2, but not each of them separately