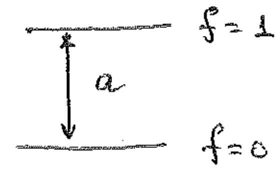
Weak magnetic field: hypefine interaction dominates

In this case we treat

$\hat{H} = \hat{H}_{coulumb} + \hat{H}_{FS} + a/\hbar^2\ \hat{S}\cdot\hat{I}$ as a unperturbed hamiltonian, with the wave function basis

$\psi_{n\ell s j I f m_f}$; $E_{n j f}$; for a given n, $j = s = 1/2$



To calculate the effect of the magnetic field, we need to calculate

$$\Delta E_{zeeman} = \langle f m_f \mid {k_1}/{\hbar}\ \hat{S}_z - {k_2}/{\hbar}\ \hat{S}_I \mid f m_f \rangle$$

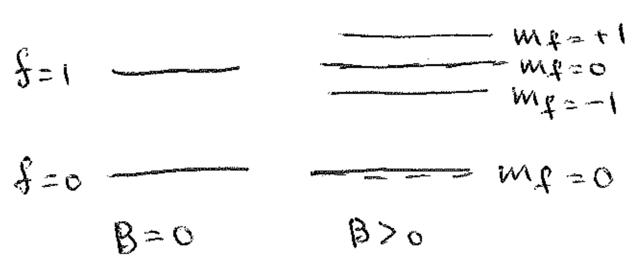
Following the same argument as in the case of the fine structure, we can state that since $\vec{S}$ and $\vec{I}$ are both precessing around $\vec{F}$ (since f and m_f provide directional information), then time-averaged values of both $\vec{S}$ and $\vec{I}$ along $\vec{F}$ are conserved, and the perpendicular component averages to zero. Thus

$$\vec{\hat{S}}_{ave} = \vec{F}\ \frac{\langle \vec{F}\cdot\vec{S} \rangle}{\langle F^2 \rangle} = \vec{F}\ \frac{\langle \hat{F}^2 \rangle + \langle \hat{S}^2 \rangle - \langle \hat{I}^2 \rangle}{\langle F^2 \rangle} = \vec{F} \quad (\text{for } s = I = 1/2)$$

and $\langle \hat{S}_{z\,ave} \rangle = \langle F_z \rangle = \hbar m_f$

Similarly $\langle \hat{I}_{z\,ave} \rangle = \langle F_z \rangle = \hbar m_f$

Thus $\Delta E_{zeeman} = (k_1 - k_2)\, m_f$



Hyperfine structure + magnetic field

$$\hat{H} = \underbrace{\hat{H}_{Coulomb} + \hat{H}_{FS}}_{\text{fixed}} + \underbrace{\frac{a}{\hbar^2}\,\hat{\vec{I}}\cdot\hat{\vec{S}} + 2\frac{\mu_B B}{\hbar}\hat{S}_z - g_p\frac{\mu_N B}{\hbar}\hat{I}_z}_{\text{perturbation hamiltonian}}$$

Fixed quantum numbers: n, $\ell=0$, $s=j=1/2$, $I=1/2$ (4x degenerate)

Strong magnetic field: we ignore the $\vec{I}\cdot\vec{S}$ term

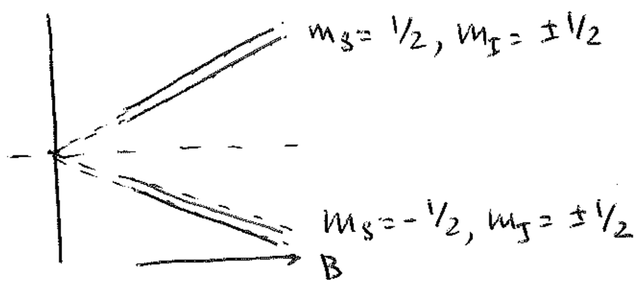
Dominating perturbation: $2\frac{\mu_B B}{\hbar}\hat{S}_z - g_p\frac{\mu_N B}{\hbar}\hat{I}_z \equiv \cancel{k_1/\hbar\,\hat{S}_z}$

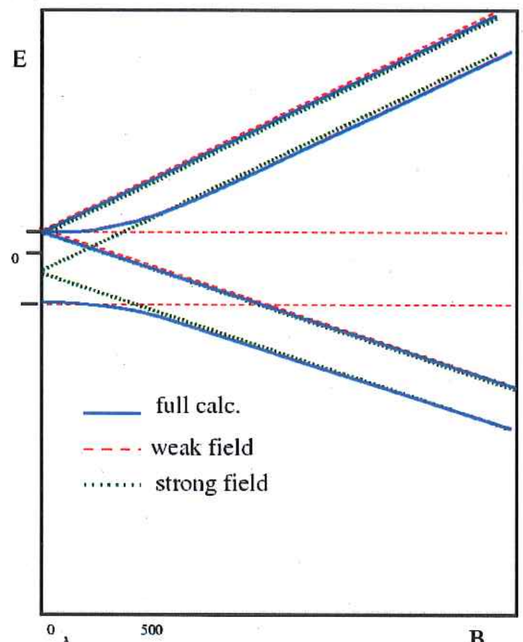
$$= k_1/\hbar\,\hat{S}_z - k_2/\hbar\,\hat{I}_z$$

(keep in mind $k_2 \ll k_1$, since $\mu_N = m_e/m_p\,\mu_B$)

Good basis to work with: $|m_s, m_I\rangle$

Energy corrections: $\Delta E_{m_s m_I} = k_1 m_s - k_2 m_I$





weak field $|f, m_f\rangle$

strong field $|m_S, m_I\rangle$