PHYS 314
Problem set # 5 *(due March 18)*
Each problem is 10 points, unless stated otherwise.

Griffiths: 9.20 (20 points)

Q1 Using explicit expressions for the wave functions for the hydrogen atom, explain which transitions are possible between $n = 1$ state and various ℓ, m-sublevels of $n = 2$ states, in case of linearly (x, z) and circularly ($\sigma_\pm$) polarized electromagnetic field ($\vec{e}_\pm = \vec{e}_x \pm i\vec{e}_y$). Assume that the light propagates along z axis, and it is also the quantization direction for the atoms.

Q2 Assume that an hydrogen atom is in the ground state at $t = \infty$. A pulsed electric field in z-direction $\mathcal{E}exp(-t^2/\tau^2)$ is applied until $t = \infty$. // Show that the total probability of the atom ending up in the $n = 2$ state is, to the first order, is:

$$P_{1\to 2}(t = +\infty) = \left(\frac{e\mathcal{E}}{\hbar}\right)^2 \left(\frac{2^{15}a^2}{3^{10}}\pi\tau e^{-\omega^2\tau^2/2}\right),$$

where $\omega = (E_2 - E_1)/\hbar$, and a is the Bohr radius.

Q3 In class we derived expression for Rabi oscillations assuming no decay of the atomic states. While usually it is impossible to properly include spontaneous emission in a wave-function description, in the case of a two-level atom the finite lifetime of the excited atomic level can be described by adding phenomenological decay terms to the probability amplitude equations:

$$\dot{c}_e = -\frac{\gamma}{2}c_e - i\frac{\Omega}{2}c_g$$
$$\dot{c}_g = -\frac{\gamma}{2}c_g - i\frac{\Omega}{2}c_e$$

where γ is the decay rate of the excited state. For an atom initially in the excited state ($c_e(t = 0) = 1$), show that the inversion (differences in populations between the excited and the ground states) is $P_e - P_g = e^{-\gamma t}cos(\Omega t)$.

Extra credit problems: These two problems deal with sudden change in the state of the system, but are not necessarily solved using the perturbation theory.

EC1: Parity measurements
A quantum system has only two energy eigestates, ψ_1 and ψ_2, corresponding to the energy eigenvalues E_1, E_2. Apart from the energy, the system is also characterized by parity, whose operator $\hat{\pi}$ acts on the energy eigenstates as follows: $\hat{\pi}|\psi_1\rangle = |\psi_2\rangle$ and $\hat{\pi}|\psi_2\rangle = |\psi_1\rangle$.
(a) Show that the eigenvalues of the parity operator are ± 1, and find the eigenstates of the parity operator in terms of $\hat{\pi}|\psi_{1,2}\rangle$.
(b) Assuming that the system is initially in a positive-parity eigenstate, find the state of the system at any time.
(c) At a particular time T a parity measurement is made on the system. What is the probability of finding the system with positive parity?
(d) *Quantum Zeno effect*: Imagine that instead of a single measurement at time T you make a series of N parity measurements at the times Δt, $2\Delta t \ldots N\Delta t = T$. Assuming that N is very large and $\Delta t \ll (E_2 - E_1)/\hbar$, what is the probability of finding the system with positive parity at time T? Compare this probability with the probability to find the system in the positive parity state with a single measurement at $t = T$. Such "freezing" of the system in the initial state for a repeated series of measurements has been called the quantum Zeno effect.

EC2: Moving potential well
An infinitely deep quantum well of width L is moving with a constant speed v along the x-axis as shown below.
(a) Find wave functions and corresponding energies of a particle of mass m in such a potential. Verify that your answer is a solution of the Schrodinger equation.
(b) Suppose that at time $t = 0$ the potential well instantaneously comes to stop. Assuming that the particle was in the ground state of the moving potential well, write down the expression for the probability of finding it in the k^{th} state of the stationary well. *You don't have to evaluate the final integrals.*
(c) Assign a condition for the smallness of the well velocity, such that the particle most likely does not change its quantum state after the well stops. Give some intuitive physical explanation for your answer.