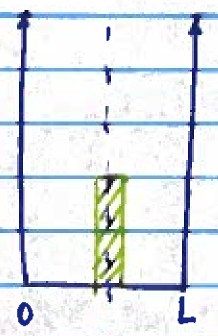
Example (to go with Mathematica demo)



$$\psi_n = \sqrt{\frac{2}{L}} \sin \frac{\pi n x}{L} \qquad E_n = \frac{\pi^2 n^2 \hbar^2}{2mL^2}$$

Initial particle distribution:
step of width a in the center

$$\psi(t=0) = \begin{cases} A & \frac{L}{2} - \frac{a}{2} < x < \frac{L}{2} + \frac{a}{2} \\ 0 & \text{elsewhere} \end{cases}$$

Normalization $\displaystyle\int_{\frac{L}{2}-a/2}^{\frac{L}{2}+a/2} |\psi(x,t=0)|^2 dx = A^2 \cdot a = 1 \Rightarrow A = 1/\sqrt{a}$

Decomposition $\displaystyle\psi(x,t=0) = \sum_{n=1}^{\infty} c_n \psi_n = \sum_{n=1}^{\infty} c_n \sqrt{\frac{2}{L}} \sin \frac{\pi n x}{L}$

$$c_n = \int_0^L \psi(x,t=0)\, \psi_n^*(x)\, dx = \int_{\frac{L}{2}-\frac{a}{2}}^{\frac{L}{2}+\frac{a}{2}} \frac{1}{\sqrt{a}} \cdot \sqrt{\frac{2}{L}} \sin \frac{\pi n x}{L}\, dx =$$

$$= \sqrt{\frac{2}{aL}} \frac{L}{\pi n} \left[\cos\left(\frac{\pi n}{2} - \frac{\pi n a}{2L}\right) - \cos\left(\frac{\pi n}{2} + \frac{\pi n a}{2L}\right) \right] = \sqrt{\frac{2}{aL}} \frac{2L}{\pi n} \sin \frac{\pi n}{2} \sin \frac{\pi n a}{2L}$$

(note: even states don't contribute due to the symmetry!)

$$\psi(x,t=0) = \sum_{n=1}^{\infty} \frac{1}{\sqrt{a}} \frac{4}{\pi n} \sin \frac{\pi n}{2} \sin \frac{\pi n a}{2L} \sin \frac{\pi n x}{L}$$

(if you want to be fancy $n \to 2k+1$
$\sin \frac{\pi n}{2} = \sin(\pi k + \frac{\pi}{2}) = (-1)^k$)

$$\psi(x,t=0) = \sum_{k=0}^{\infty} \frac{1}{\sqrt{a}} (-1)^k \frac{4}{\pi(2k+1)} \sin \frac{\pi(2k+1)a}{2L} \times$$
$$\times \sin \frac{\pi(2k+1)x}{L}$$

Time evolution

$$\psi(x,t) = \sum_{n=1}^{\infty} c_n \psi_n(x)\, e^{-iE_n t/\hbar} =$$

$$= \sum_{n=1}^{\infty} \frac{1}{\sqrt{a}} \frac{4}{\pi n} \sin \frac{\pi n}{2} \sin \frac{\pi n a}{2L} \sin \frac{\pi n x}{L}\, e^{-iE_n t/\hbar}$$

The exact solution requires infinite number of states. The more you include, the more accurate it is