Quantum teleportation

Two-particle entangle-state basis — Bell states

$$|\Psi_{12}^{(\pm)}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle\right)$$

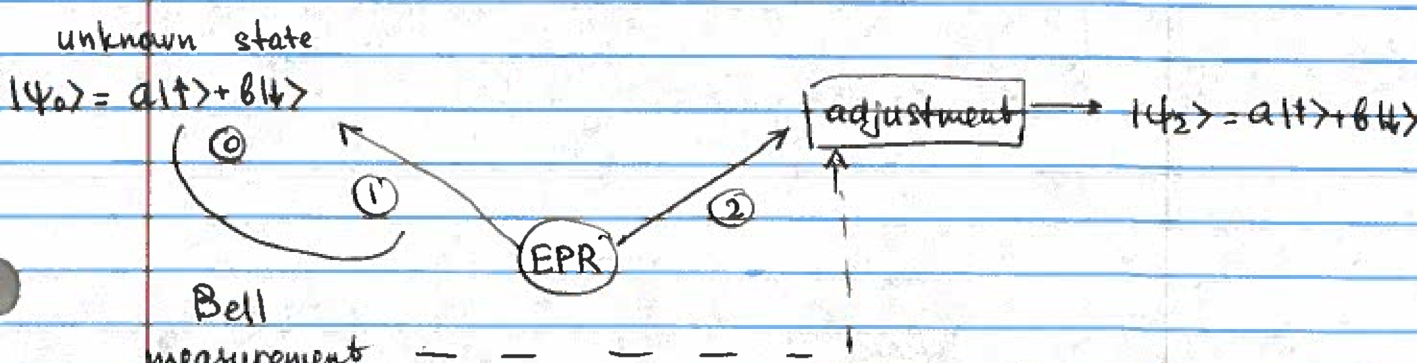
$$|\Phi_{12}^{(\pm)}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle\right)$$

The general idea behind the teleportation:

1. We need to replicate a state of a single spin-1/2 particle in a different location.

We cannot measure its state and then recreate it, as a single measurement doesn't allow to gain full information about its state

However, we can make a joint two-photon state measurement with one half of the entangled pair, such that this measurement collapses the other half to the desired state.



EPR pair in $|\Psi^-_{12}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$ state

Test particle

$$|\psi_0\rangle = a|\uparrow\rangle + b|\downarrow\rangle$$

A three particle state $|\psi_{012}\rangle = |\psi_0\rangle \otimes |\Psi^-_{12}\rangle =$

$$= (a|\uparrow\rangle + b|\downarrow\rangle)\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) =$$

$$= \frac{a}{\sqrt{2}}|\uparrow\uparrow\downarrow\rangle - \frac{a}{\sqrt{2}}|\uparrow\downarrow\uparrow\rangle + \frac{b}{\sqrt{2}}|\downarrow\uparrow\downarrow\rangle - \frac{b}{\sqrt{2}}|\downarrow\downarrow\uparrow\rangle$$

Next, we make a joint measurement (like measuring total spin of two particles) with the test particle and the particle 1

$$|\uparrow\uparrow\rangle = \frac{1}{\sqrt{2}}(\Phi^+_{01} + \Phi^-_{01}) \qquad |\downarrow\downarrow\rangle = \frac{1}{\sqrt{2}}(\Phi^+_{01} - \Phi^-_{01})$$

$$|\uparrow\downarrow\rangle = \frac{1}{\sqrt{2}}(\Psi^+_{01} + \Psi^-_{01}) \qquad |\downarrow\uparrow\rangle = \frac{1}{\sqrt{2}}(\Psi^+_{01} - \Psi^-_{01})$$

$$= |\Phi^+_{01}\rangle\frac{1}{\sqrt{2}}\left(\frac{a}{\sqrt{2}}|\downarrow\rangle - \frac{b}{\sqrt{2}}|\uparrow\rangle\right) + |\Phi^-_{01}\rangle\frac{1}{2}(+a|\downarrow\rangle + b|\uparrow\rangle) +$$

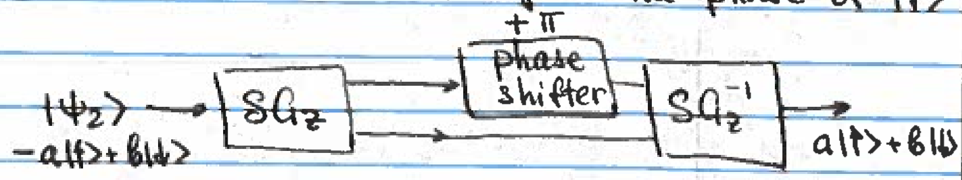
$$+ |\Psi^+_{01}\rangle\frac{1}{2}(-a|\uparrow\rangle + b|\downarrow\rangle) + |\Psi^-_{01}\rangle\frac{1}{2}(-a|\uparrow\rangle - b|\downarrow\rangle)$$

If we measure... then $|\psi_2\rangle =$

$|\Psi^-_{01}\rangle$ $\qquad$ $|\psi_2\rangle = a|\uparrow\rangle + b|\downarrow\rangle$ target

$|\Psi^+_{01}\rangle$ $\qquad$ $|\psi_2\rangle = -a|\uparrow\rangle + b|\downarrow\rangle$

need to change the phase of $|\uparrow\rangle$



if we measure then

$|\Phi^-_{01}\rangle$ $\qquad$ $|\psi_2\rangle = a|\downarrow\rangle + b|\uparrow\rangle$

need to flip both spins

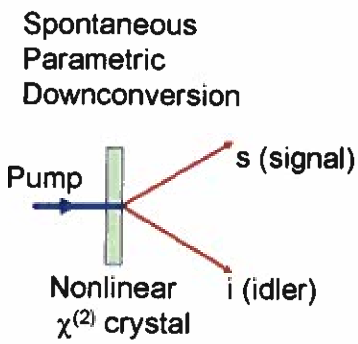
$$\underset{b|\uparrow\rangle + a|\downarrow\rangle}{|\psi_2\rangle} \longrightarrow \boxed{\text{NMR } \pi\text{-pulse}} \longrightarrow \underset{\text{target}}{b|\downarrow\rangle + a|\uparrow\rangle}$$

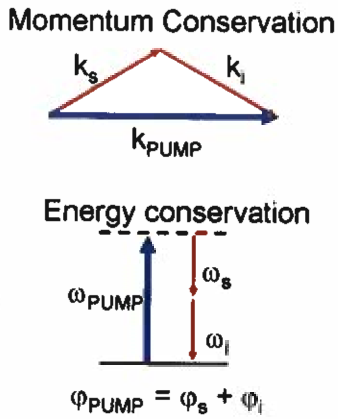
$|\Phi^+_{01}\rangle$ $\qquad$ $|\psi_2\rangle = +a|\downarrow\rangle - b|\uparrow\rangle$

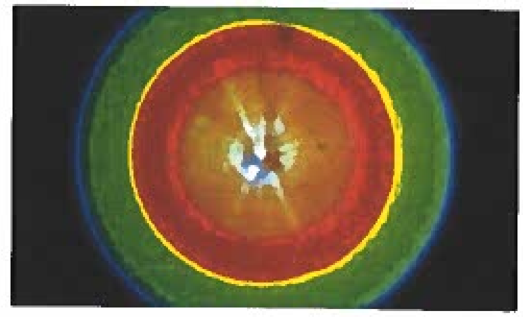
need to do both spin-flip and the phase adjustment

In any case, one can produce the copy of the original unknown state at the output
QM allows that since we never gain much direct information about a & b, so we did not destroy it in the process of measurements.

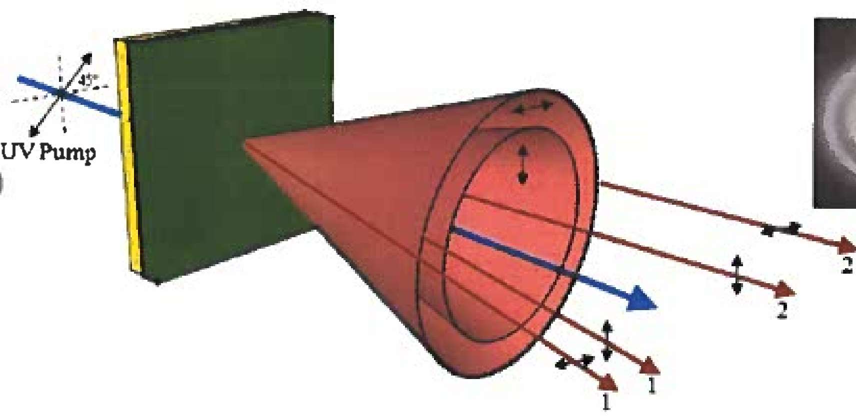
Source of correlated single photons

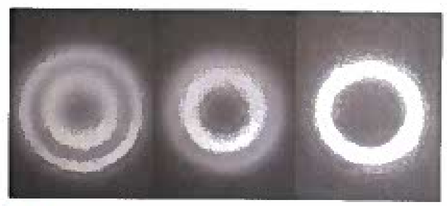




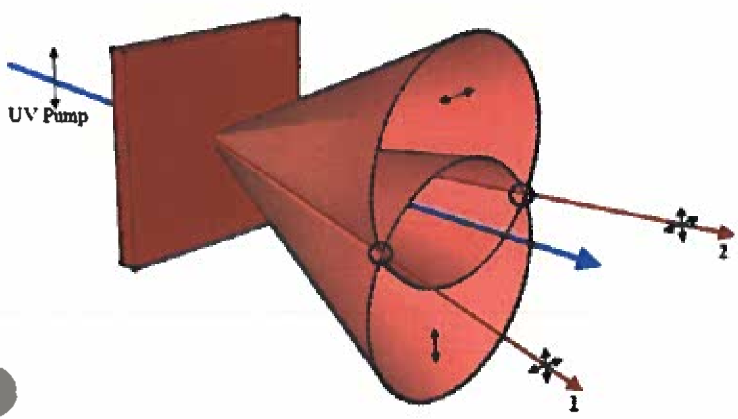


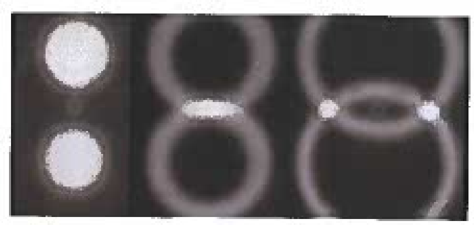
Polarization-entangled states





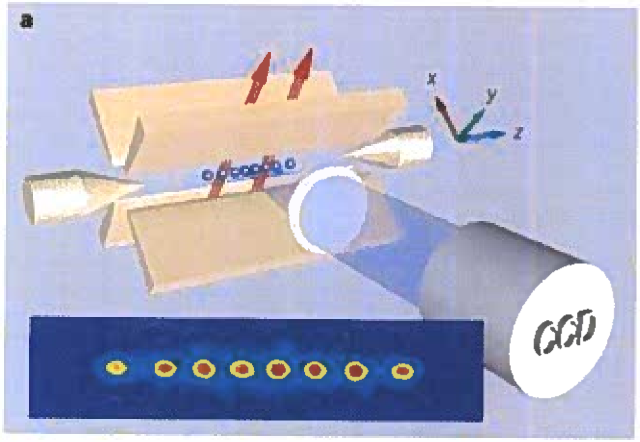
$$|H\rangle_1 |H\rangle_2 \pm e^{i\varphi} |V\rangle_1 |V\rangle_2$$

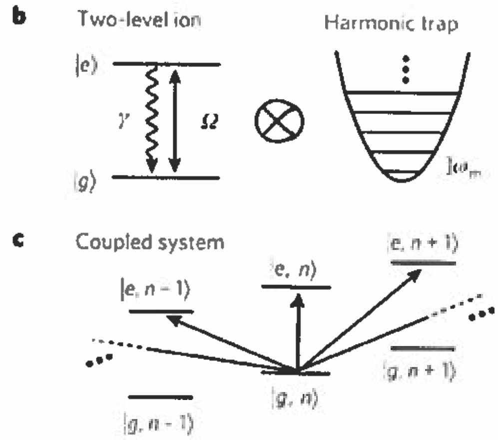




$$|H\rangle_1 |V\rangle_2 \pm e^{i\varphi} |V\rangle_1 |H\rangle_2$$

Entangled ions → collective motion





Entanglement b/w photon polarization and atom/ion spin