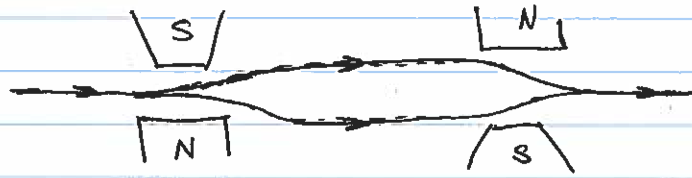
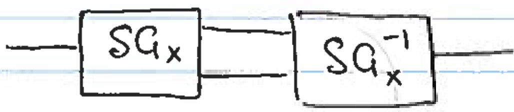
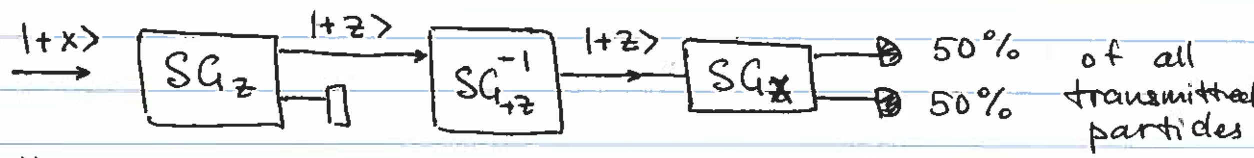
Which way ~~information~~ witchery and quantum interference

Stern-Gerlach loop





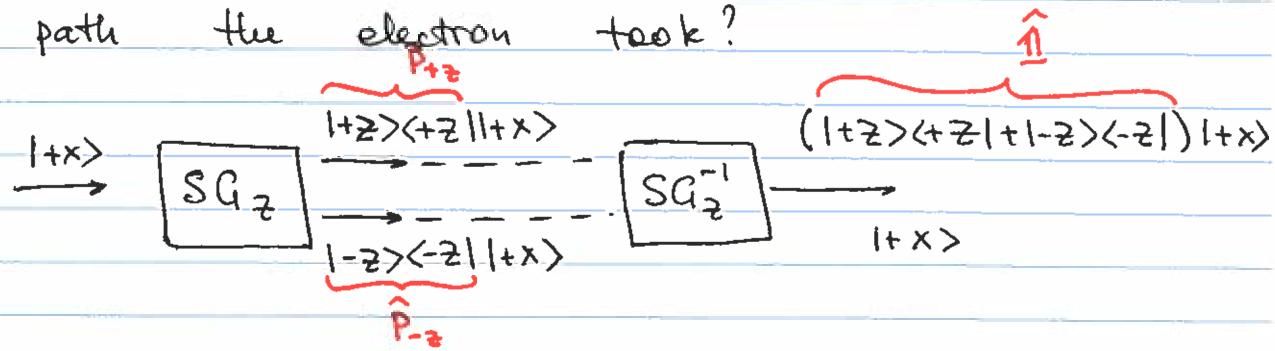
"Experimental" data



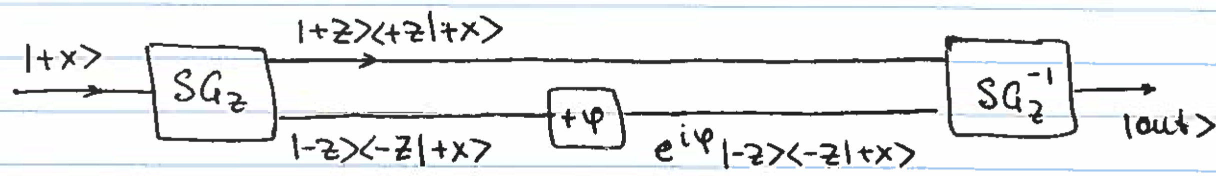
Here we know that we detected only the particle that took upper path in the loop. Same thing if the top path is blocked, we know that all the particles came from the bottom path.

Note that if an electron is measured after SG_z to be in either $|+z\rangle$ or $|-z\rangle$ states and then released along the same path, it will arrive at the SG_x analyzer either in $|+z\rangle$ or $|-z\rangle$ state, so when analyzed they will go to either $|+x\rangle$ or $|-x\rangle$ detector with 50% / 50% probability

What if we cannot know which path the electron took?



Can we build an interferometer?



We can add a phase using a phase shift gate

$$\hat{P}(\varphi)\,|\psi\rangle = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{pmatrix} |\psi\rangle \quad \text{in } |z\rangle \text{ basis}$$

$$\hat{P}(\varphi)\begin{pmatrix} \langle+z|+x\rangle \\ \langle-z|+x\rangle \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{pmatrix}\begin{pmatrix} \langle+z|+x\rangle \\ \langle-z|+x\rangle \end{pmatrix} = \begin{pmatrix} \langle+z|+x\rangle \\ e^{i\varphi}\langle-z|+x\rangle \end{pmatrix}$$

$$|out\rangle = |+z\rangle\underbrace{\langle+z|+x\rangle}^{1/\sqrt{2}} + e^{i\varphi}\,|-z\rangle\underbrace{\langle-z|+x\rangle}^{1/\sqrt{2}} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ e^{i\varphi} \end{pmatrix}$$

The probability to measure the output in state $|+x\rangle$:

$$|\langle+x|out\rangle|^2 = |\underbrace{\langle+x|+z\rangle}_{1/\sqrt{2}}\underbrace{\langle+z|+x\rangle}_{1/\sqrt{2}} + e^{i\varphi}\underbrace{\langle+x|-z\rangle}_{1/\sqrt{2}}\underbrace{\langle-z|+x\rangle}_{1/\sqrt{2}}|^2 = \frac{1}{4}|1+e^{i\varphi}|^2$$

$$= \frac{1}{4}|e^{-i\varphi/2} + e^{i\varphi/2}|^2 = \cos^2\varphi/2$$

$$|\langle-x|out\rangle|^2 = |\underbrace{\langle-x|+z\rangle}_{1/\sqrt{2}}\underbrace{\langle+z|+x\rangle}_{1/\sqrt{2}} + e^{i\varphi}\underbrace{\langle-x|-z\rangle}_{-1/\sqrt{2}}\underbrace{\langle-z|+x\rangle}_{1/\sqrt{2}}|^2 = \frac{1}{4}|1-e^{i\varphi}|^2$$

$$= \frac{1}{4}|e^{i\varphi/2} - e^{-i\varphi/2}| = \sin^2\varphi/2$$

Same calculations using matrix notation

$$\langle +x|out\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\end{pmatrix}\begin{pmatrix}1\\ e^{i\varphi}\end{pmatrix}\frac{1}{\sqrt{2}} = \frac{1}{2}(1+e^{i\varphi}) = \frac{e^{i\varphi/2}(e^{i\varphi/2}+e^{-i\varphi/2})}{2}$$

$$= e^{i\varphi/2}\cos\varphi/2$$

$$|\langle +x|out\rangle|^2 = \cos^2\varphi/2$$

$$\langle -x|out\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & -1\end{pmatrix}\begin{pmatrix}1\\ e^{i\varphi}\end{pmatrix}\frac{1}{\sqrt{2}} = \frac{1}{2}(1-e^{i\varphi}) = -ie^{i\varphi/2}\ \frac{e^{i\varphi/2}-e^{-i\varphi/2}}{2}$$

$$= \frac{1}{i}e^{i\varphi/2}\sin\varphi/2$$

$$|\langle -x|out\rangle|^2 = \sin^2\varphi/2$$

For $\varphi=0$ (no added shift) $P_{+x}=1$ $P_{-x}=0$

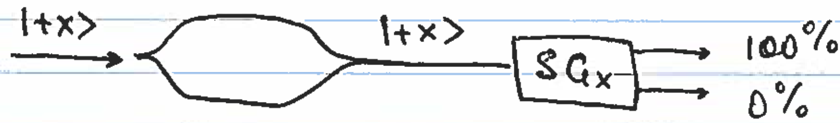
For $\varphi=\pi$ $P_{+x}=0$ $P_{-x}=1$! We managed to flip spin's direction

Spooky action on a distance?
Non-interactive measurements

Can we detect something without being there?

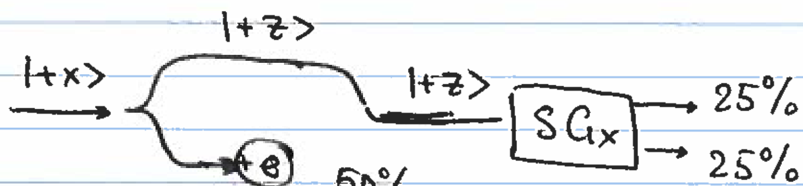
Imagine someone dropped a positron in one of the loop's path. Can we possibly notice with one measurement?
Not always, but actually yes!

No positron



← no clicks ever in |-x> output

With positron



total annihilation!

So in 50% no $\bar{e}$ emerges (positron is there, but it is too late)

In 25% the $\bar{e}$ emerges from $|+x\rangle$ — no useful information

BUT

In 25% the $\bar{e}$ emerges from $|-x\rangle$ — we detected +e without even getting close!