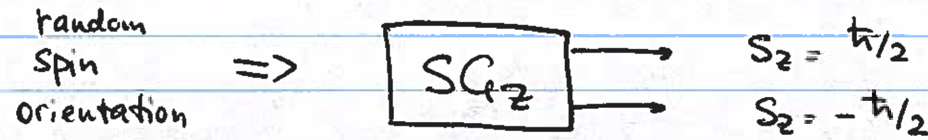
Quantum state vectors

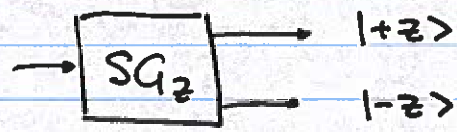


We are going to label these states

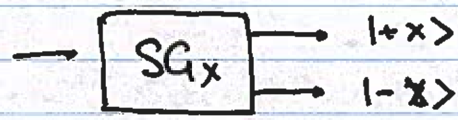
if a particle emerges on the top path (with $S_z = \hbar/2$) : $|+z\rangle$ or $|\uparrow\rangle$ (spin up)

if it emerges on the lower path (with $S_z = -\hbar/2$) : $|-z\rangle$ or $|\downarrow\rangle$ (spin down)

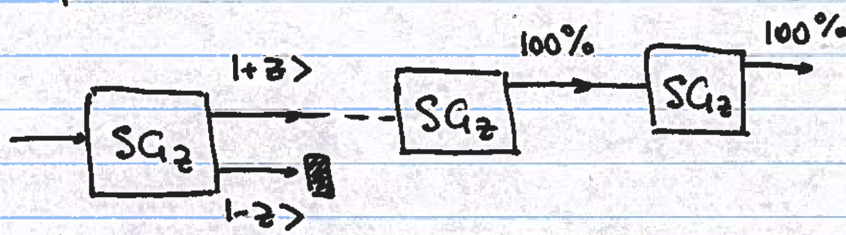
So



and

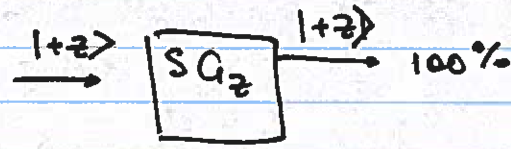


Important:

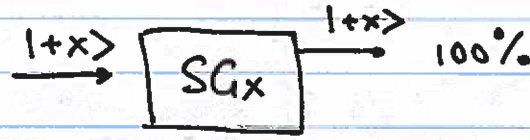


A particle in the state $|+z\rangle$ will always emerge from the top output;
and a particle in the state $|-z\rangle$ → from the bottom

<u>Thus</u>: measuring $|\pm z\rangle$ ~~is~~ using SG_z does not change the state: it is an <u>eigenstate</u> of this operation.

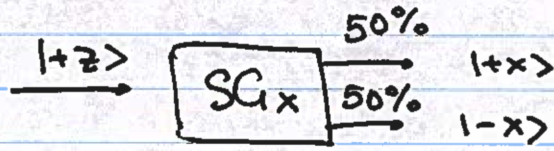


eigenstate

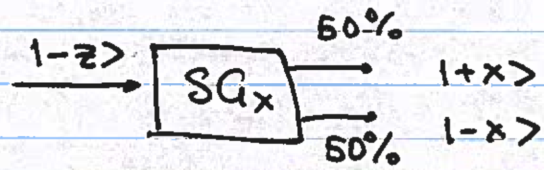


eigentate

However

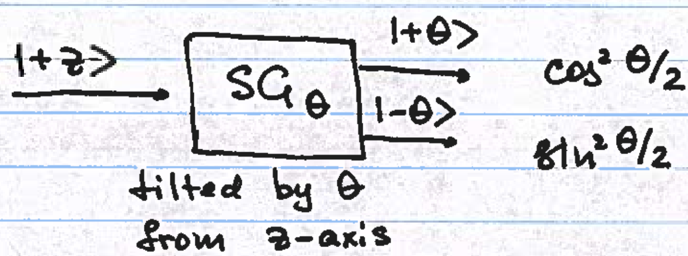


not an eigentate! the output state differs from the input state



(The outcome of S_x measurement is random)

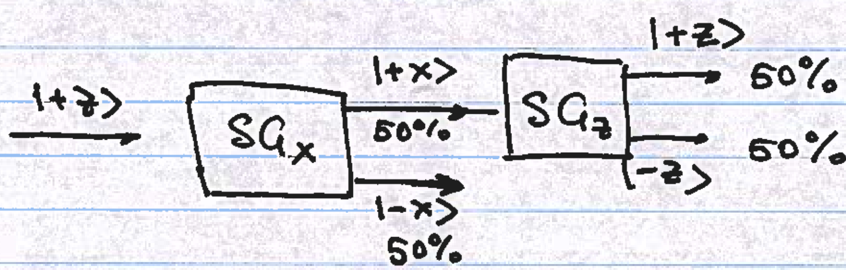
In general



The closer θ is to 0 the more predictable the outcome becomes

In reality, we can~~not~~ obtain certain information only about one component of the atomic spin.

Measuring one component erases information about others



Weirdness of quantum spins (or quantum bits)

We are used to think about spins ~~to~~ as vectors in 3D space

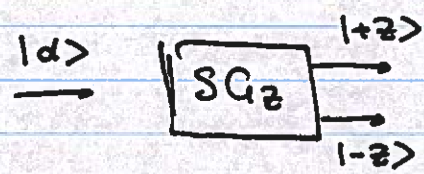
$$\vec{S} = S_x \hat{i} + S_y \hat{j} + S_z \hat{k}$$

That requires measuring all three components independently → impossible in QM!

Instead, spins live in a binary (2D) space, and the eigenstates for one components can be expressed as linear combination of the other!

Let's choose the basis of $|\pm z\rangle$ we then can express any other states corresponding to various orientation as a combination of these two states

$$|\alpha\rangle = c_+ |+z\rangle + c_- |-z\rangle$$



Probability $|c_+|^2$

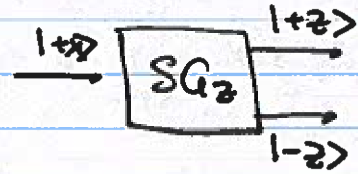
" $|c_-|^2$

c_+ and c_- can be complex numbers

$|c_+|^2$ and $|c_-|^2$ are real <u>non-negative</u> numb (absolute values)

Since a particle must exit somewhere with 100% probability

$|c_+|^2 + |c_-|^2 = 1$ normalization

Since 

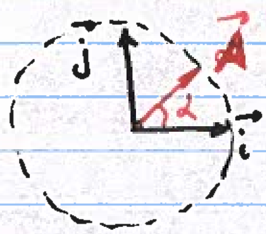
50% $\quad |c_+|^2 = 1/2 \quad |c_+| = \frac{1}{\sqrt{2}}$

50% $\quad |c_-|^2 = 1/2 \quad |c_-| = \frac{1}{\sqrt{2}}$

We will see that

$$|+x\rangle = \frac{1}{\sqrt{2}}|+z\rangle + \frac{1}{\sqrt{2}}|-z\rangle$$
$$|-x\rangle = \frac{1}{\sqrt{2}}|+z\rangle - \frac{1}{\sqrt{2}}|-z\rangle$$
$$|+y\rangle = \frac{1}{\sqrt{2}}|+z\rangle + \frac{i}{\sqrt{2}}|-z\rangle$$
$$|-y\rangle = \frac{1}{\sqrt{2}}|+z\rangle - \frac{i}{\sqrt{2}}|-z\rangle$$

This is somewhat similar to 2D vector decomposition



$$\vec{A} = \underbrace{\cos\alpha}_{c_i} \cdot \vec{i} + \underbrace{\sin\alpha}_{c_j} \vec{j} = c_i \vec{i} + c_j \vec{j}$$

$$c_i = \vec{A}\cdot\vec{i} \quad \& \quad c_j = \vec{A}\cdot\vec{j}$$
$$|A|^2 = \vec{A}\cdot\vec{A}$$

To introduce similar "dot product"-like operation, we need to introduce a <u>bra</u> vector

ket $|\alpha\rangle \longrightarrow$ bra $\langle\alpha|$

$\langle$ bra $|$ ket $\rangle$

$$|\alpha\rangle = c_+ |+z\rangle + c_- |-z\rangle$$
$$\langle\alpha| = c_+^* \langle +z| + c_-^* \langle -z|$$

Inner product $\langle\alpha|\beta\rangle$ is the probability amplitude for a particle in the state $|\beta\rangle$ to be found in the state $|\alpha\rangle$ (analog of a scalar product for vectors)

$\langle\alpha|\alpha\rangle = 1$ "length" of the state vector is always unity

Orthogonal states $\to$ $\langle\alpha|\beta\rangle = 0$

States $|+z\rangle$ and $|-z\rangle$ are orthogonal

$$\langle +z|-z\rangle = 0$$

$$\langle +x|-x\rangle = 0 \quad \text{and} \quad \langle +y|-y\rangle = 0$$

If $|\alpha\rangle = c_+|+z\rangle + c_-|-z\rangle$

then $\langle +z|\alpha\rangle = c_+ \underbrace{\langle +z|+z\rangle}_{=1} + c_- \underbrace{\langle +z|-z\rangle}_{=0} = c_+$

$\langle -z|\alpha\rangle = c_+ \underbrace{\langle -z|+z\rangle}_{=0} + c_- \underbrace{\langle -z|-z\rangle}_{=1} = c_-$

The probability of detecting a particle in the $|+z\rangle$ state $\quad P_+ = |c_+|^2 = |\langle +z|\alpha\rangle|^2$

in the $|-z\rangle$ state $\quad P_- = |c_-|^2 = |\langle -z|\alpha\rangle|^2$

Normalization check $\langle\alpha|\alpha\rangle = 1$

$\langle\alpha| = c_+^* \langle +z| + c_-^* \langle -z|$

$|\alpha\rangle = c_+|+z\rangle + c_-|-z\rangle$

$$\langle\alpha|\alpha\rangle = c_+ c_+^* \langle +z|+z\rangle + \underbrace{\cancel{c_+^* c_- \langle +z|-z\rangle}}_{=0} + \underbrace{\cancel{c_+ c_-^* \langle -z|+z\rangle}}_{=0} + c_- c_-^* \langle -z|-z\rangle =$$

$$= |c_+|^2 + |c_-|^2 = 1$$