Basics of Quantum Computing

QM textbook	QC textbook
$\|\alpha\rangle$ quantum state	$\|\alpha\rangle$ qubit
basis states	binary states
$\|+z\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ $\|-z\rangle = \begin{pmatrix}0\\1\end{pmatrix}$	$\|0\rangle$ "0" $= \begin{pmatrix}1\\0\end{pmatrix}$, $\|1\rangle$ "1" $= \begin{pmatrix}0\\1\end{pmatrix}$
Operators	1-qubit gate
$\hat{A}\|\alpha\rangle = \|\beta\rangle$	$\xrightarrow{\text{input}} \boxed{A} \xrightarrow{\text{output}}$

Common quantum gate

Qubit gates are <u>always</u> unitary (i.e. reversable $A^{-1} = A^{\dagger}$)

Pauli gates

$-\boxed{X}-$ $\hat{\sigma}_x$ $\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$ ← aka NOT gate

$-\boxed{Y}-$ $\hat{\sigma}_y$ $\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$

$-\boxed{Z}-$ $\hat{\sigma}_z$ $\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$

$-\boxed{I}-$ $\hat{\mathbb{1}}$ $\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

input	output
0	1
1	0

$|0\rangle = \hat{\sigma}_x |1\rangle$

$|1\rangle = \hat{\sigma}_x |0\rangle$

Hadamar gate

$-\boxed{H}-$ $\hat{H} = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$

$\hat{H}|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1/\sqrt{2}\\ 1/\sqrt{2}\end{pmatrix}$

$\hat{H}|1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1/\sqrt{2}\\ -1/\sqrt{2}\end{pmatrix}$

Phase gate

$-\boxed{S}-$ $\hat{S} = \begin{pmatrix}1 & 0\\ 0 & i\end{pmatrix}$

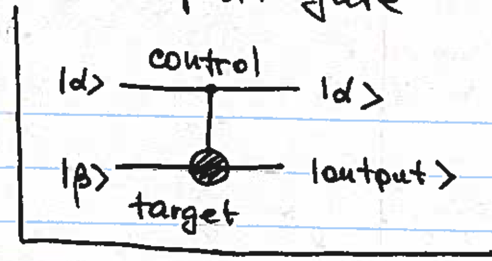
$\hat{S}|0\rangle = |0\rangle$

$\hat{S}|1\rangle = i|1\rangle$

$\pi/8$ gate

$-\boxed{T}-$ $\hat{T} = \begin{pmatrix}1 & 0\\ 0 & e^{i\pi/4}\end{pmatrix} = e^{i\pi/8}\begin{pmatrix}e^{-i\pi/8} & 0\\ 0 & e^{i\pi/8}\end{pmatrix}$

two qubit-gate

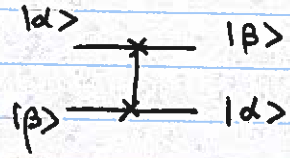


Multiqubit gates

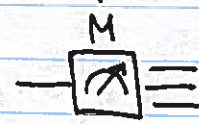
CNOT gate = control NOT

control	target
$|0\rangle$	unchanged
$|1\rangle$	flip target

Swap gate



Measurement operations cannot be a part of a quantum circuit, since they are not reversible.
They are applied at the end to the final output state



probabilistic quantum bits

2.11 The postulates of quantum mechanics

- *States.* States of physical systems are represented by vectors in Hilbert spaces. This postulate says that a physical state in a quantum system can be represented as one of the vectors $|.\rangle$ in the Dirac notation defined above.
- *Observables.* Observables are represented by Hermitian operators. This is because these operators have real eigenvalues, which are appropriate for representing physical quantities (such as an amount of energy, or a distance from the Sun, for example).
- *Measurement.* A quantum state can be measured by use if a set of orthogonal projections. If $|\phi_1\rangle, \ldots, |\phi_k\rangle$ are orthogonal states, then a quantum state $|\psi\rangle$ can be measured by use of $|\phi_1\rangle, \ldots, |\phi_k\rangle$ and collapses into the state $|\phi_i\rangle$ with probability $|\langle\phi_i|\psi\rangle|^2$.
- *Unitary Evolution.* Any change that takes place in a quantum system which is not a measurement can be expressed by the action of a unitary operation.

from "Introduction to Quantum Information Science" by V. Vedral