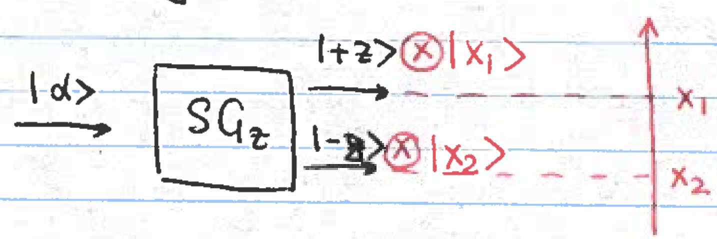
Quantum entanglement

Entanglement is a property of two or more quantum systems which exhibit correlations that cannot be explained by ~~the~~ classical physics.

We actually already introduce a sort of entanglement is SG apparatus analysis



In this measurement we entangled atomis spin component with atom's position

Quantum state afte SG_z: $c_1|+z\rangle \otimes |x_1\rangle + c_2|-z\rangle \otimes |x_2\rangle$

Screen – a ~~projection~~ x measurement operator revealing if a particle is in $x = x_1$ or $x = x_2$, and from that we know the particle spin

Here by taking a measurement of one parameter we gain instant knowledge of the other

Two spin system:

$|+z\rangle_1 |-z\rangle_2$ – separable (not entangled) state there is no correlation b/w the measurements of the two particles.

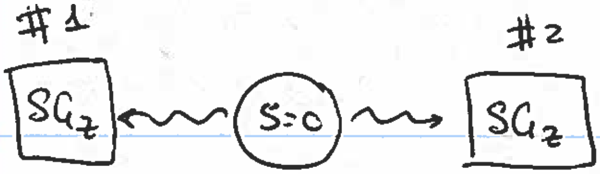
Let's consider a compound $S=0$ particle

~~since~~ ~~[illegible]~~

Momentum conservation

$\vec{S}_1 + \vec{S}_2 = \vec{S} = 0$

$\vec{S}_1 = -\vec{S}_2$



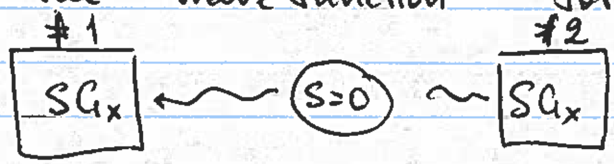
(from previous lectures)

$|\alpha_{12}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

if $S_{z_1} \to |\uparrow\rangle$ then $S_{2z} \to |\downarrow\rangle$ $\qquad |\uparrow\downarrow\rangle = |\uparrow\rangle_1 \otimes |\downarrow\rangle_2$

if $S_{z_1} \to |\downarrow\rangle$ then $S_{2z} \to |\uparrow\rangle$ $\qquad |\downarrow\uparrow\rangle = |\downarrow\rangle_1 \otimes |\uparrow\rangle_2$

However, we will get the same form of the wave function for <u>any</u> basis



$|\alpha_{12}\rangle = \frac{1}{\sqrt{2}}(|+x,-x\rangle - |-x,+x\rangle)$

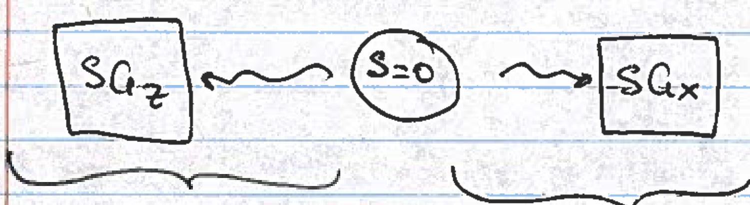
if $|+x\rangle_1 \longrightarrow$ then $|-x\rangle_2$

$|-x\rangle_1 \longrightarrow$ then $|+x\rangle_2$

Thus, if we measure one particle in some basis with a certain value of spin, we are guaranteed to detect the other particle in the same basis with the opposite spin.

The Einstein - Podolsky - Rosen Paradox

Such states (often referred to as EPR pair) seem to allow violation of an uncertainty principle!



provides information about $\hat{S}_z$ for <u>both</u> particles

provides information about $\hat{S}_x$ for <u>both</u> particles

???

Einstein argued this is a proof that QM is not a complete theory

Resolution: the act of the first measurement collapses the entangled state!

Original state $|d_{12}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$ or $\frac{1}{\sqrt{2}}\left(|+x,-x\rangle - |-x,+x\rangle\right)$

If particle #1 is measured in ~~the~~ the z-basis and, say, found in the state $|\uparrow\rangle_1$, we effectively apply the P_+ projection operator

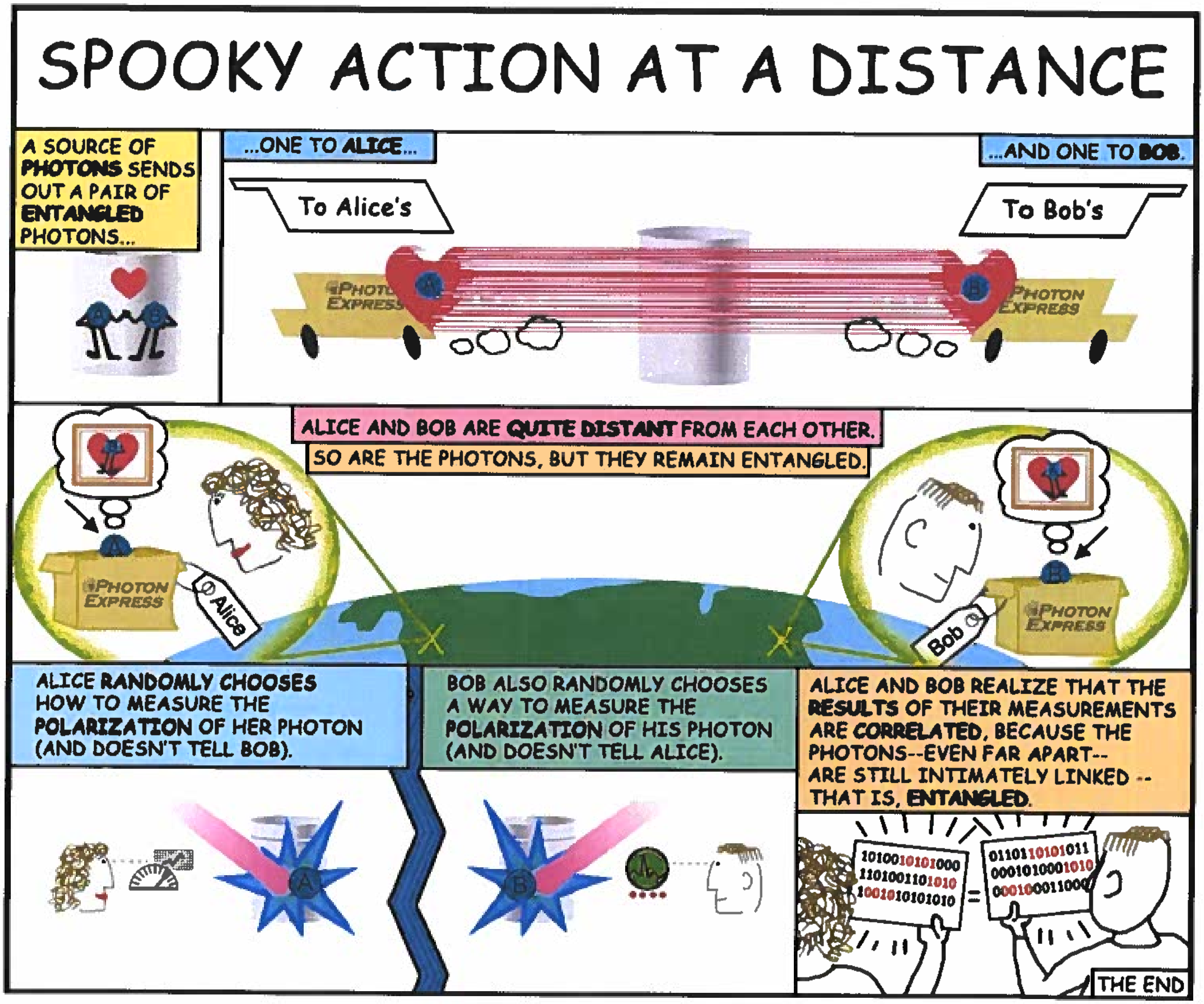
$$P_+ |d_{12}\rangle = |\uparrow\rangle_1 \langle\uparrow|_1 \cdot \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_1 \otimes |\downarrow\rangle_2 - |\downarrow\rangle_1 \otimes |\uparrow\rangle_2 \right) =$$

$$= \frac{1}{\sqrt{2}} |\uparrow\rangle_1 \underline{\underline{|\downarrow\rangle_2}}$$ – non-entangled state! single-particle state

Now spin 2 state is known $|\downarrow\rangle_2$, so an attempt to pass it through SG_x will not provide any information about x-component of spin for the first particle. and outputs $|\pm x\rangle_2$ will have 50% probability

But the contraversy remains — non-locality. A measurement of a state for one particle must <u>instantly</u> collapse the state of the other independently of their distance → can be faster than ~~the~~ light ?!

Partial resolution: even though the state collapse may be instantaneous, ~~the~~ one cannot gain any extra information from it faster than the speed of light, since the correlation can only be observed when both outcomes are compared, and such classical communication is only possible at the speed of light (or slower)


SPOOKY ACTION AT A DISTANCE
A SOURCE OF PHOTONS SENDS OUT A PAIR OF ENTANGLED PHOTONS...
...ONE TO ALICE...
...AND ONE TO BOB.
To Alice's
To Bob's
PHOTON EXPRESS
PHOTON EXPRESS
ALICE AND BOB ARE QUITE DISTANT FROM EACH OTHER.
SO ARE THE PHOTONS, BUT THEY REMAIN ENTANGLED.
PHOTON EXPRESS
Alice
Bob
PHOTON EXPRESS
ALICE RANDOMLY CHOOSES HOW TO MEASURE THE POLARIZATION OF HER PHOTON (AND DOESN'T TELL BOB).
BOB ALSO RANDOMLY CHOOSES A WAY TO MEASURE THE POLARIZATION OF HIS PHOTON (AND DOESN'T TELL ALICE).
ALICE AND BOB REALIZE THAT THE RESULTS OF THEIR MEASUREMENTS ARE CORRELATED, BECAUSE THE PHOTONS--EVEN FAR APART--ARE STILL INTIMATELY LINKED -- THAT IS, ENTANGLED.
THE END

Bell's inequality

If we assume that the hidden-variable theory is valid, then for three possible measurement orientations two particles must be in 8 possible states

	1	2
1:	$\{+a, +b, +c\}$	$\{-a, -b, -c\}$
2:	$\{+a, +b, -c\}$	$\{-a, -b, +c\}$
3:	$\{+a, -b, +c\}$	$\{-a, +b, -c\}$
4:	$\{+a, -b, -c\}$	$\{-a, +b, +c\}$
5:	$\{-a, +b, +c\}$	$\{+a, -b, -c\}$
6:	$\{-a, +b, -c\}$	$\{+a, -b, +c\}$
7:	$\{-a, -b, +c\}$	$\{+a, +b, -c\}$
8:	$\{-a, -b, -c\}$	$\{+a, +b, +c\}$

$$P(+a,+b) = P_3 + P_4$$
$$P(+a,+c) = P_2 + P_4$$
$$P(+c,+b) = P_3 + P_7$$

$$P(+a,+b) = P_3 + P_4 \leq P(+a,+c) + P(+c,+b) =$$
$$= P_3 + P_4 + \underbrace{P_2 + P_7}_{\geq 0}$$

If we can identify the conditions for which $P(+a,+b) > P(+a,+c) + P(+c,+b)$ we would have proven that HVT is not valid.

Let's look at QM treatment

$$|-n\rangle \longleftarrow (s=0) \longrightarrow \boxed{SG_{\vec{n}}}$$

$|+n\rangle = \cos\frac{\theta}{2}|+z\rangle + \sin\frac{\theta}{2}|-z\rangle$
with 50% chance

$|-n\rangle = \sin\frac{\theta}{2}|+z\rangle + \cos\frac{\theta}{2}|-z\rangle$

θ angle
$\varphi = 0$

$$P(\underset{\uparrow\atop a}{+\theta},\underset{\uparrow\atop b}{+0}) = \frac{1}{2}\cdot\sin^2\frac{\theta}{2}$$

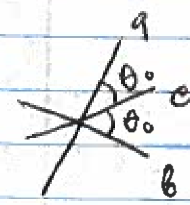
Since rotating the apparatus must not change anything

$$P(+a,+b) = \frac{1}{2}\sin^2\frac{\theta_{ab}}{2}$$

$$P(+a,+b) = \frac{1}{2}\sin^2\frac{\theta_{ab}}{2}$$

$$P(+a,+c) + P(+c,+b) = $$
$$= \frac{1}{2}\sin^2\frac{\theta_{ac}}{2} + \frac{1}{2}\sin^2\frac{\theta_{bc}}{2}$$

If $\theta_{ac} = \theta_{bc} = \theta_0$ and $\theta_{ab} = 2\theta_0$

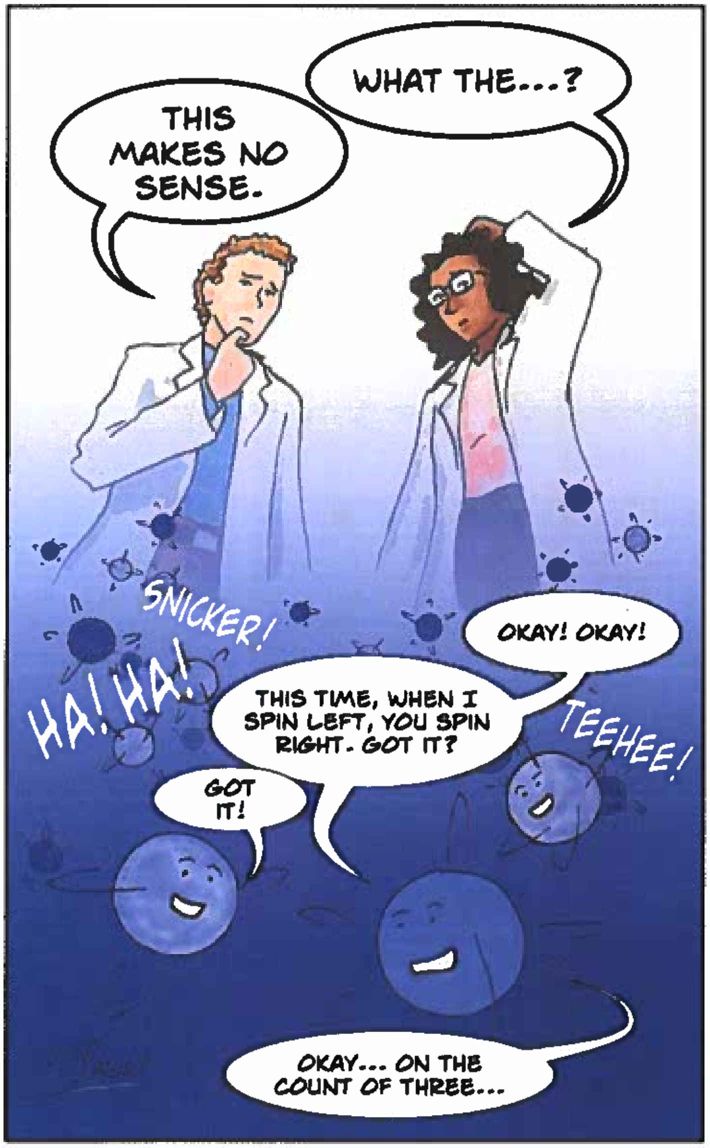


$$\sin^2\theta_0 \quad \text{vs} \quad 2\sin^2\theta_0/2$$
$$4\cancel{\sin^2\theta/2}\cos^2\theta/2 \quad \text{vs} \quad 2\cancel{\sin^2\theta/2}$$

$$\cos^2\theta/2 \quad \text{vs} \quad 1/2$$

for $\theta/2 \not< \pi/4$

Bell's inequality is violated


THIS MAKES NO SENSE.
WHAT THE...?
SNICKER!
HA! HA!
OKAY! OKAY!
THIS TIME, WHEN I SPIN LEFT, YOU SPIN RIGHT. GOT IT?
TEEHEE!
GOT IT!
OKAY... ON THE COUNT OF THREE...