Quantum information lingvo

In quantum mechanics we talk about states and operators that act on the quantum state

In quantum information they talk about qubits and gates → but these are the same things!

QM	QI
State $\|\psi\rangle$ (arbitrary)	$\|\psi\rangle$ qubit
Basis state (for example)	Basis states "0" & "1"
$\|+z\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ & $\|-z\rangle = \begin{pmatrix}0\\1\end{pmatrix}$	$\|0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ & $\|1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$
$\|\psi\rangle = c_1\|+z\rangle + c_2\|-z\rangle$	$\|\psi\rangle = c_1\|0\rangle + c_2\|1\rangle$
Operator $\hat{A}$ $\quad \|out\rangle = \hat{A}\|\psi\rangle$	Gate (1-qubit gate)
	$\|out\rangle = \hat{A}\|\psi\rangle$
	$\|\psi\rangle \rightarrow [\hat{A}] \rightarrow \|out\rangle$

Common gates / operators

Their Matrices

- X — (aka ⊕) — X-gate or NOT-gate
- Y — Y-gate
- Z — Z-gate

$$\hat{\sigma}_x = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} \qquad \hat{\sigma}_y = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} \qquad \hat{\sigma}_z = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

Pauli gates

- H — Hadamard gate

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} \qquad \left[\hat{H}|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right]$$

- φ — Phase shift gate

$$\begin{pmatrix}1 & 0\\ 0 & e^{i\varphi}\end{pmatrix}$$