WILLIAM & MARY

CHARTERED 1693

QUANTUM STATES AS VECTORS

09/08/2023

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Vectors

Review

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$$\vec{V} = V_x \hat{i} + V_y \hat{j} + V_z \hat{k}$$

- where: V_x, V_y, V_z are real numbers
- $\hat{i}$, $\hat{j}$, $\hat{k}$: is an orthogonal basis

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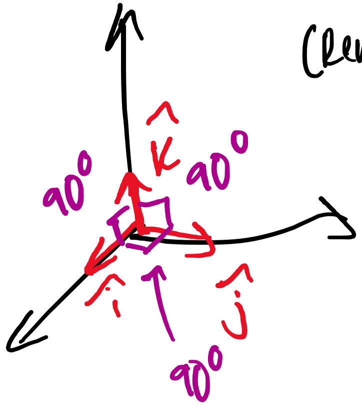

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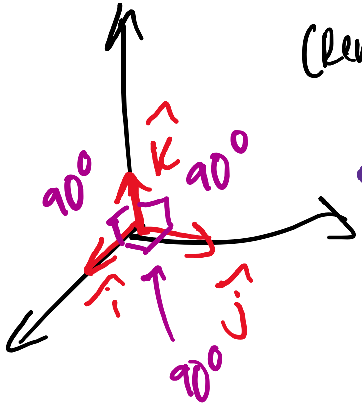

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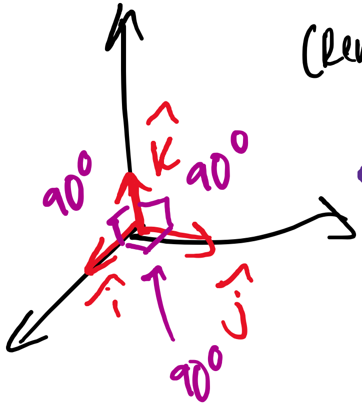

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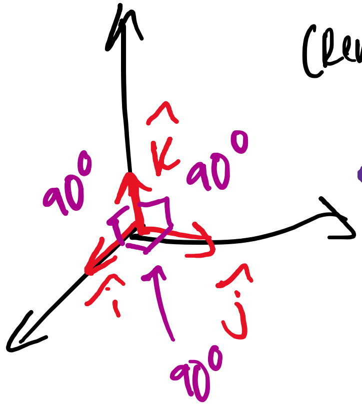

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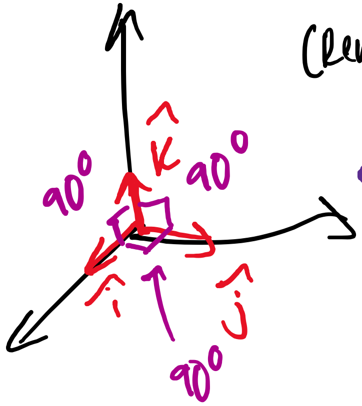

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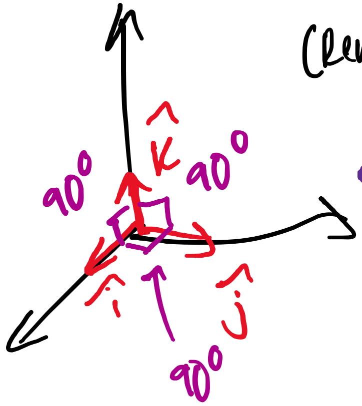

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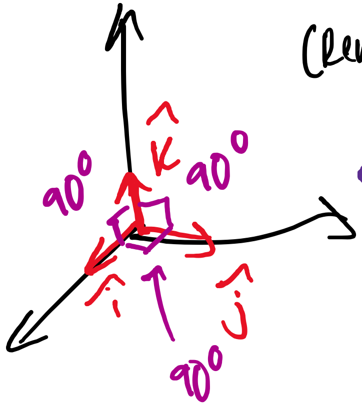

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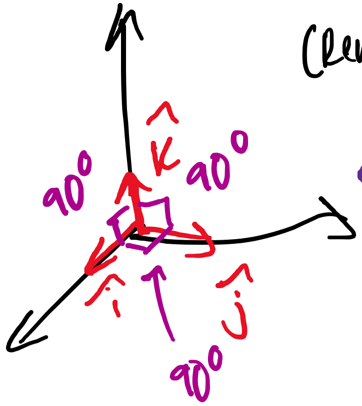

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The magnitude of the Quantum state is always 1

Probability is normalized

Vectors

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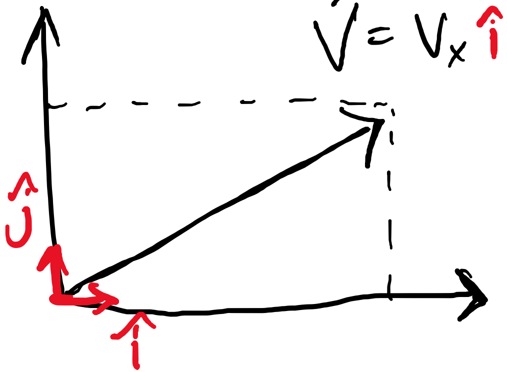

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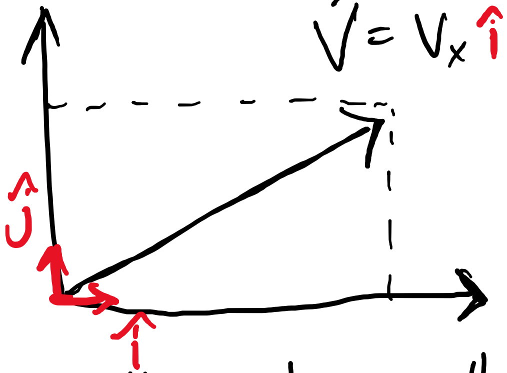

Projection of the vector in the $\hat{i}$ direction?

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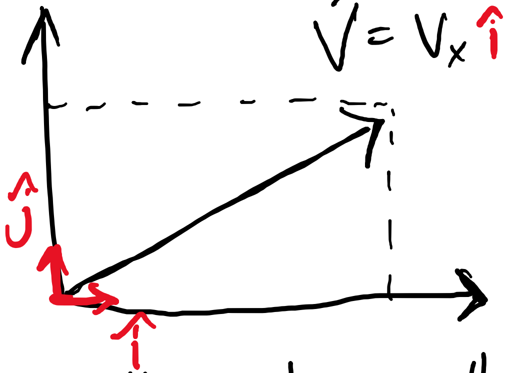

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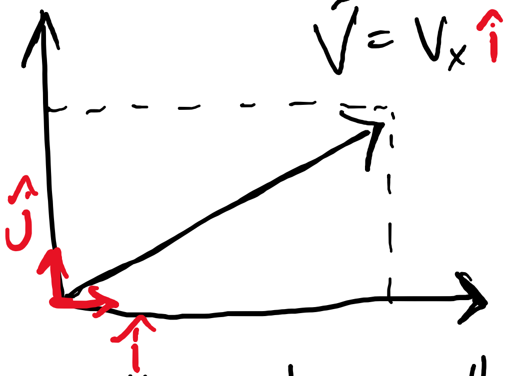

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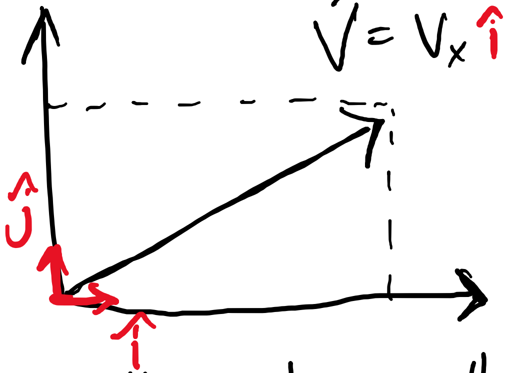

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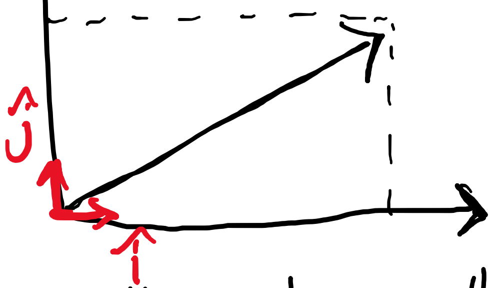

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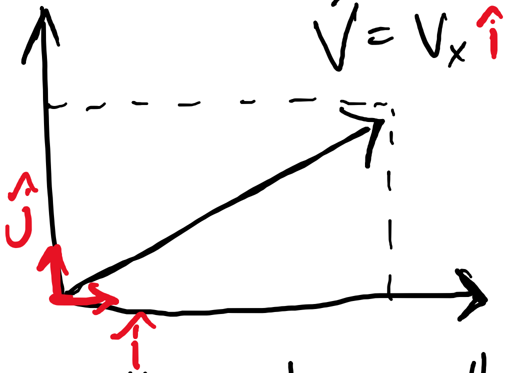

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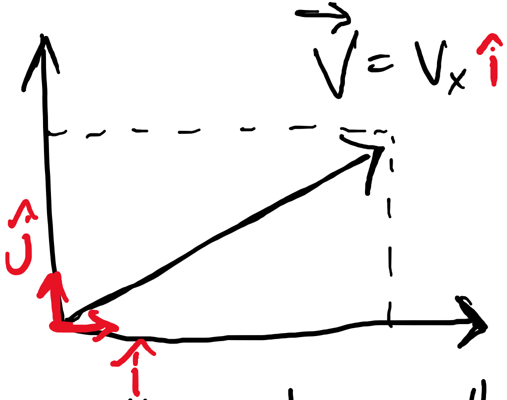

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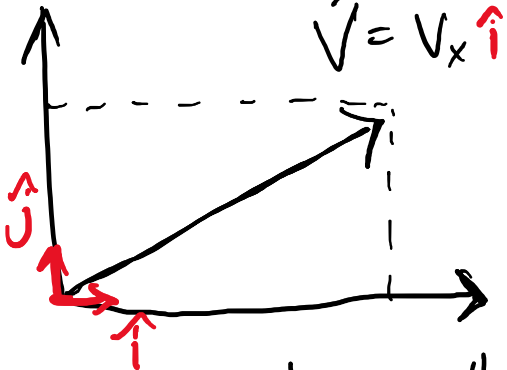

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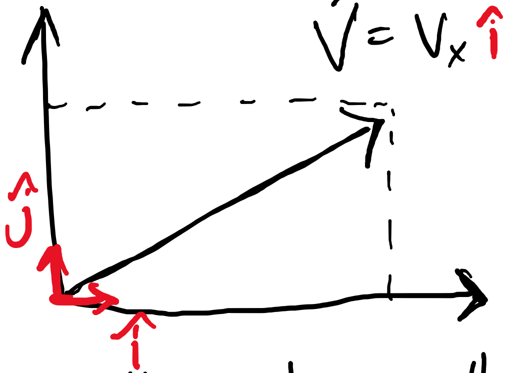

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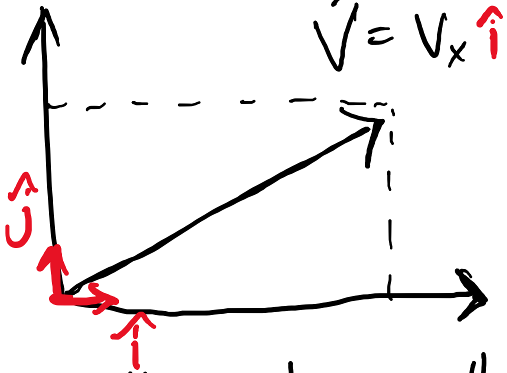

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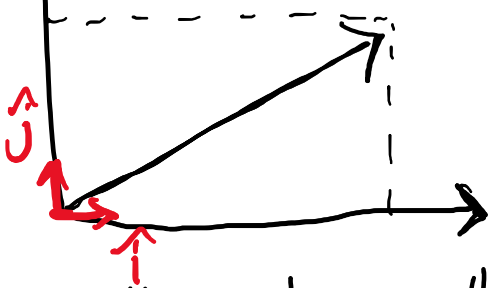

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☝️🤓 what we measure in the experiments is

Act. 1 $|\langle b|\psi\rangle|^2$ ← this is the probability of finding the particle in state $|b\rangle$ in the experiment

Example: A particle in a superposition of spin up and down states

$$|\psi\rangle = \frac{1}{2}|+z\rangle + i\,\frac{\sqrt{3}}{2}|-z\rangle$$

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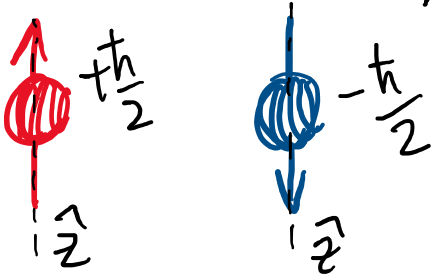

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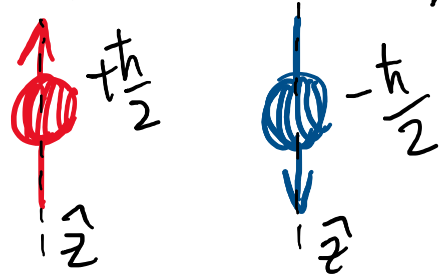

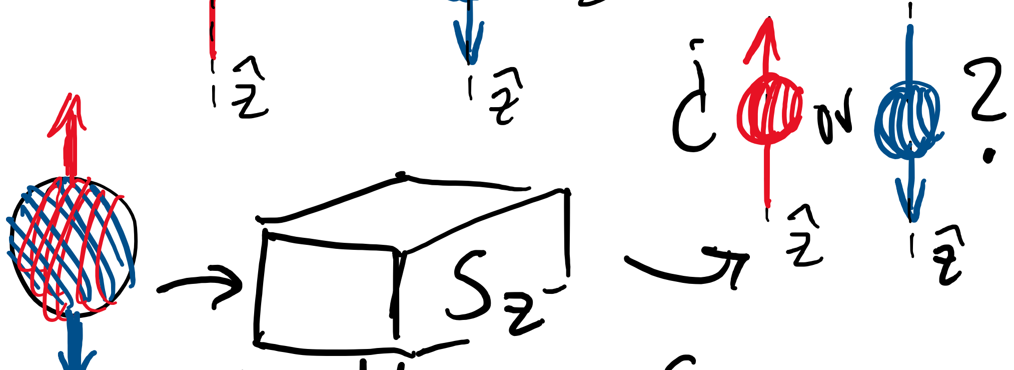

experiment to measure S_z

Example: A particle in a superposition of spin up and down states

$$|\psi\rangle = \frac{1}{2}|+z\rangle + i\,\frac{\sqrt{3}}{2}|-z\rangle$$

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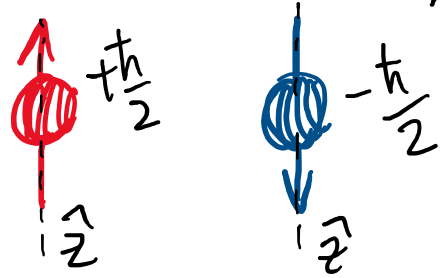

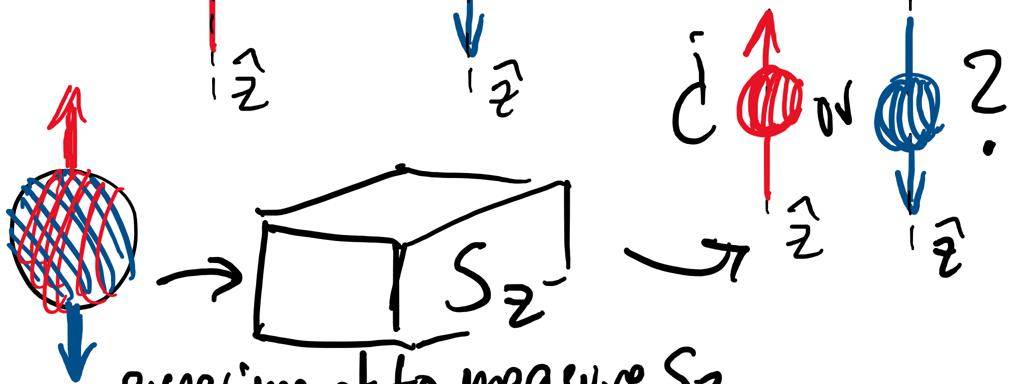

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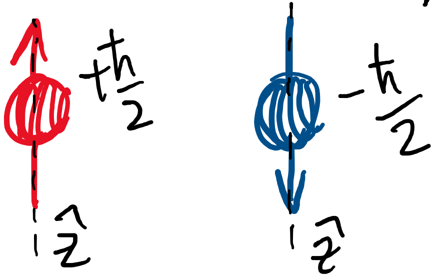

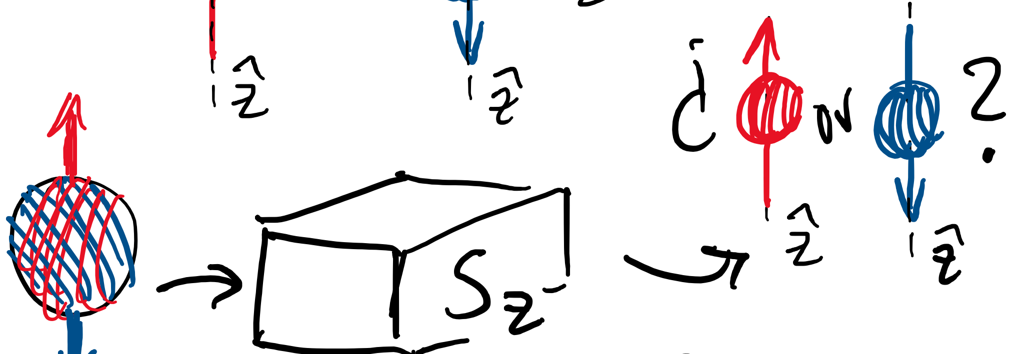

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Impossible to know (this example)

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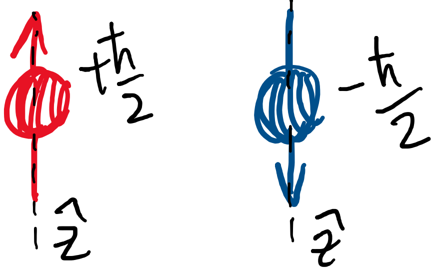

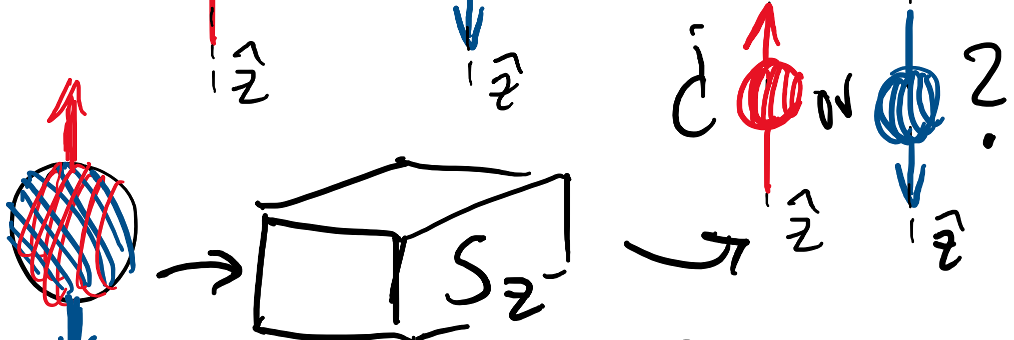

experiment to measure S_z

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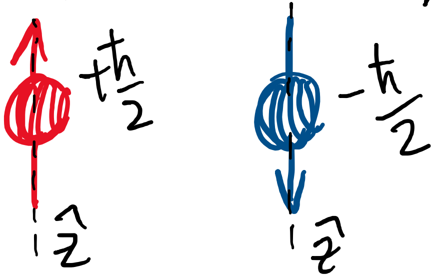

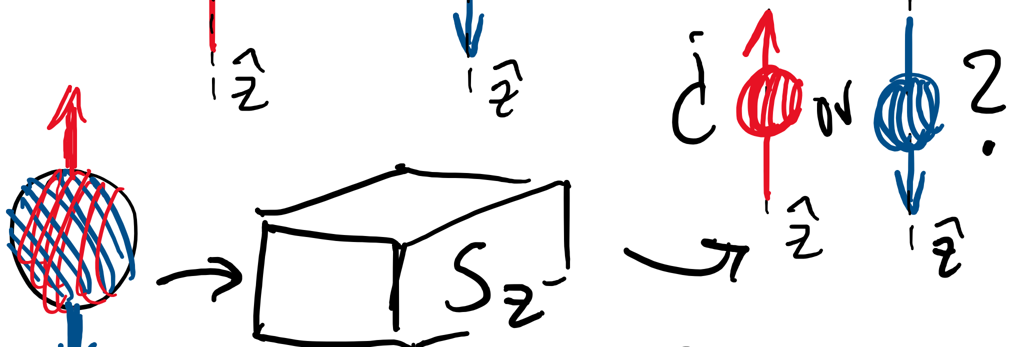

experiment to measure S_z

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I. Calculate the probability of measuring 

II. Calculate the probability of measuring 

$$|\psi\rangle = \frac{1}{2}|+z\rangle + i\,\frac{\sqrt{3}}{2}|-z\rangle$$

I. Calculate the probability of measuring the particle in the state $|+z\rangle$; $P_+ = |\langle +z|\psi\rangle|^2$

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first:

$$\langle +z|\psi\rangle = \frac{1}{2}\langle +z\cancel{|+z\rangle}^{\,1} + i\frac{\sqrt{3}}{2}\cancel{\langle +z|-z\rangle}^{\,0}$$

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then $P_+ = |\langle +z|\psi\rangle|^2 = \frac{1}{4}$

there is a 25% probability of obtaining the "spin up" ($S_z = +\frac{\hbar}{2}$, $\uparrow$)

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1) Do the same with $|-z\rangle$

or

2) $P_+ + P_- = 1$ (probability is normalized)

then $P_- = 1 - P_+ = 1 - \frac{1}{4}$

$$P_- = \frac{3}{4}$$

the probability of obtaining spin down ($S_z = -\frac{\hbar}{2}$, $\downarrow$) is 75%

$$|\psi\rangle = \frac{1}{2}|+z\rangle + i\frac{\sqrt{3}}{2}|-z\rangle$$

I. Calculate the probability of measuring the particle in the state $|+z\rangle$; $P_+ = |\langle +z|\psi\rangle|^2$

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II. Calculate the probability of measuring the particle in the state $|-z\rangle$, two options

1) Do the same with $|-z\rangle$

lets do it the other way

$|\langle -z|\psi\rangle|^2$; first $\langle -z|\psi\rangle = i\frac{\sqrt{3}}{2}$

then $$\left|i\frac{\sqrt{3}}{2}\right|^2 = \underbrace{\left(i\frac{\sqrt{3}}{2}\right)^*}_{\text{complex conjugate}}\left(i\frac{\sqrt{3}}{2}\right)$$

$$= \left(-i\frac{\sqrt{3}}{2}\right)\left(i\frac{\sqrt{3}}{2}\right)$$

$$|\langle -z|\psi\rangle|^2 = \frac{3}{4}$$

Expectation Value and uncertainty

$$|\chi\rangle = \frac{1}{2}|+z\rangle + i\,\frac{\sqrt{3}}{2}|-z\rangle$$

- what is the spin value (along $\hat{z}$) of the particle?
- Can we know it by performing one measurement?

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Best I can do is the Average

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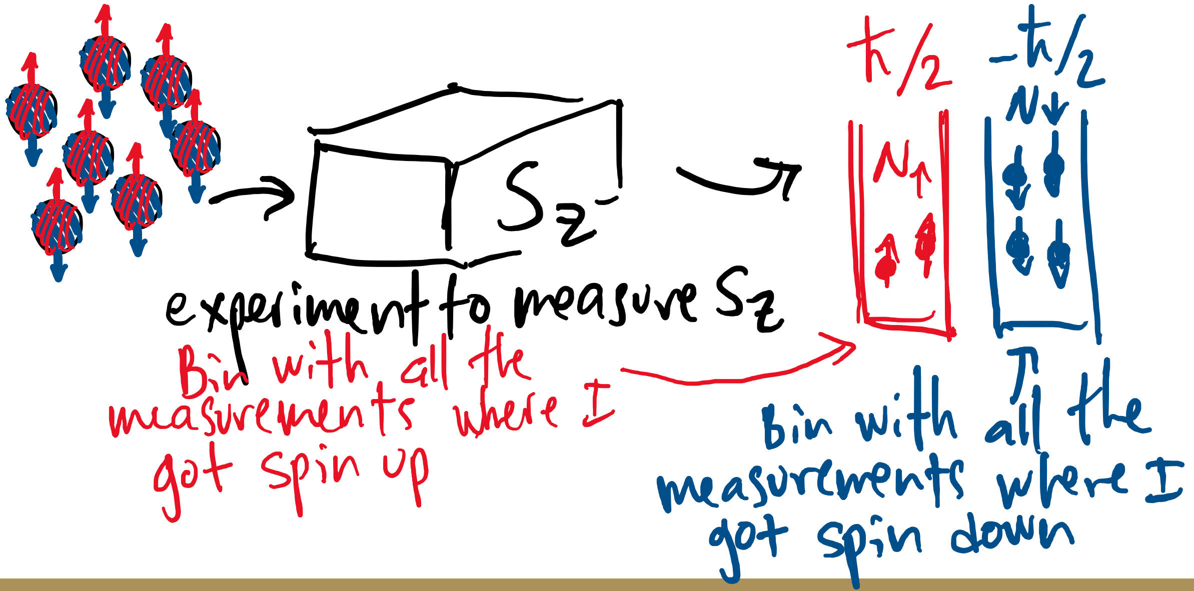
if I repeat the experiment many times, what is the average of the output's results?



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$$|\psi\rangle = \frac{1}{2}|+z\rangle + i\,\frac{\sqrt{3}}{2}|-z\rangle$$

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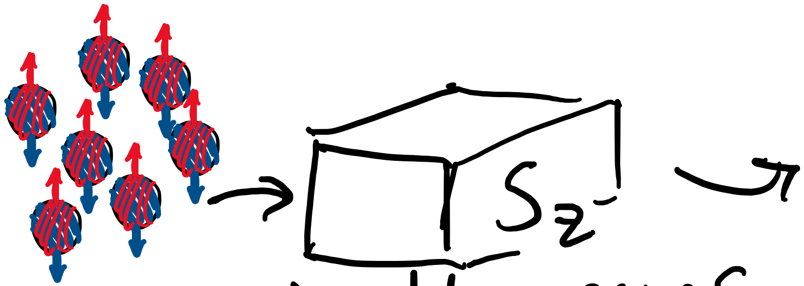
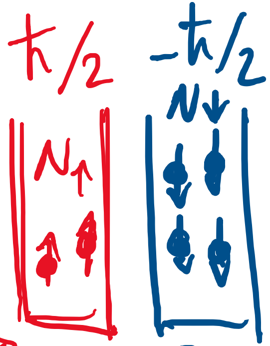
$$\text{Average} = \frac{1}{N_T}\left[N_\uparrow\left(\frac{\hbar}{2}\right) + N_\downarrow\left(-\frac{\hbar}{2}\right)\right]$$



experiment to measure S_z



Bin with all the measurements where I got spin up

Bin with all the measurements where I got spin down

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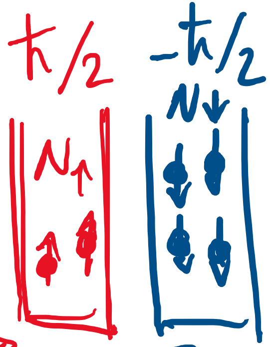


Bin with all the measurements where I got spin up

Bin with all the measurements where I got spin down

$$\underbrace{|\langle +z|\psi\rangle|^2\left(\frac{\hbar}{2}\right) + |\langle -z|\psi\rangle|^2\left(-\frac{\hbar}{2}\right)}$$

this average is called Expectation Value

$$\langle S_z\rangle$$

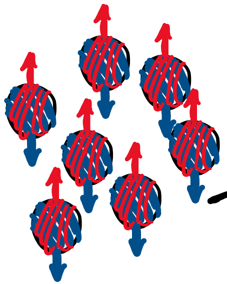
- Calculate the expectation value of S_z with $|\psi\rangle$

Expectation Value and uncertainty

$$|\psi\rangle = \frac{1}{2}|+z\rangle + i\frac{\sqrt{3}}{2}|-z\rangle$$

if I repeat ... times

what is ...



experiment to measure S_z

Bin with all the measurements where I got spin up

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Average $= \frac{1}{N_T}\left[N_\uparrow\left(\frac{\hbar}{2}\right) + N_\downarrow\left(-\frac{\hbar}{2}\right)\right]$

$\ldots\left(\frac{\hbar}{2}\right) + |\langle -z|\psi\rangle|^2\left(-\frac{\hbar}{2}\right)$

...erage is called ...tation value

$\langle S_z\rangle$

- Calculate the expectation value of S_z with $|\psi\rangle$

☝🤓 Act. II

The expectation value IS NOT the most probable value of a measurement

$$|\psi\rangle = \frac{1}{2}|+z\rangle + i\,\frac{\sqrt{3}}{2}|-z\rangle$$

$$\langle S_z\rangle = |\langle +z|\psi\rangle|^2\left(\frac{\hbar}{2}\right) + |\langle -z|\psi\rangle|^2\left(-\frac{\hbar}{2}\right)$$

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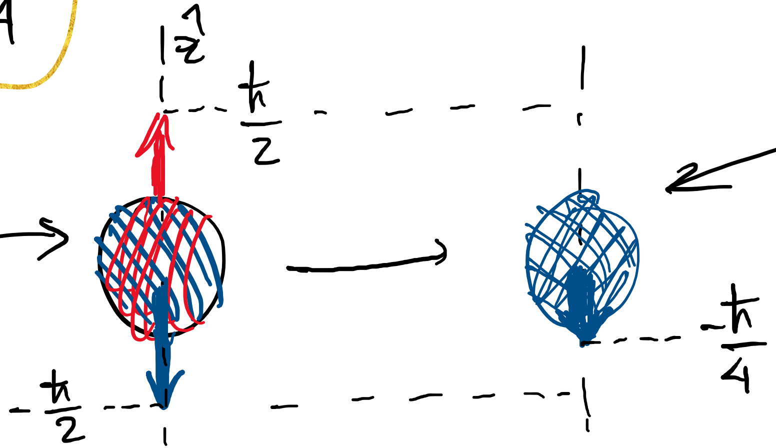
We do not measure the superposition



what we measure is the expectation value of an observable