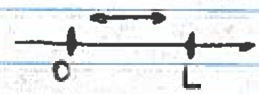
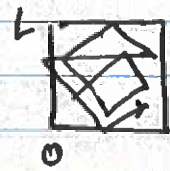
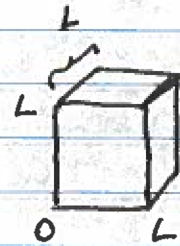
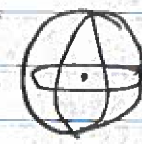
Infinite square well in the worlds of different dimentions

1D 

2D  or  3D  or 

In each case a particle moves freely inside the allowed region

$$\hat{H} = \frac{\hat{p}^2}{2m} \; ; \text{ in Cartesian } x\text{-basis} \quad \hat{H} = -\frac{\hbar^2}{2m}\left(\underbrace{\underbrace{\underbrace{\frac{\partial^2}{\partial x^2}}_{1D}+\frac{\partial^2}{\partial y^2}}_{2D}+\frac{\partial^2}{\partial z^2}}_{3D}\right)$$

in spherical coordinates $\hat{H} = -\frac{\hbar^2}{2m}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{\hat{L}^2}{2\mu r^2}$

Boundary conditions $x_i = 0$ & $x_i = L$ $\psi(\vec{x}) = 0$

The geometry of a boundary determines the symmetry of the solution

Ractangular boundaries $\longrightarrow$ Cartesian coordinates
Spherically sym boundaries $\longrightarrow$ spherical coordinates

Cartesian coordinates $\longrightarrow$ Schrodinger equation

1D: $-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi \Rightarrow \frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi = 0$

$\psi(x) = A\cos kx + B\sin kx \qquad k = \sqrt{\frac{2mE}{\hbar^2}} = P/\hbar$

Boundary conditions $\psi(0) = \psi(L) = 0$

$\psi_n(x) = B\sin\frac{\pi n x}{L} \Rightarrow \sqrt{\frac{2}{L}}\sin\frac{\pi n x}{L} \qquad k_n = \frac{\pi n}{L}$

2D: $-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi(x,y)}{\partial x^2} + \frac{\partial^2\psi(x,y)}{\partial y^2}\right) = E\psi$

Separation of variables (since x- and y- motions are independent)

$\psi(x,y) = X(x)\cdot Y(y)$

$-\frac{\hbar^2}{2m}\left(\frac{d^2X}{dx^2}\cdot Y + X\frac{d^2Y}{dy^2}\right) = E X\cdot Y \qquad \cdot\frac{1}{X\cdot Y}$

$$\underbrace{\frac{1}{X}\frac{d^2X}{dx^2}}_{\text{depends only on } X} + \underbrace{\frac{1}{Y}\frac{d^2Y}{dy^2}}_{\text{depends only on } Y} + \underbrace{\frac{2mE}{\hbar^2}}_{\text{constant}} = 0$$

$-C_1 \leftarrow$ must be constant $\rightarrow -C_2$

$-\frac{2mE_x}{\hbar^2} = -\frac{p_x^2}{2m} \qquad -\frac{2mE_y}{\hbar^2} = -\frac{p_y^2}{2m}$

$\frac{1}{X}\frac{d^2X}{dx^2} + \frac{2mE_x}{\hbar^2} = 0 \qquad \frac{1}{Y}\frac{d^2Y}{dy^2} + \frac{2mE_y}{\hbar^2} = 0$

$X'' + k_x^2 X = 0 \qquad Y'' + k_y^2 Y = 0$

Two independent 1D motions

$X_n(x) = \sqrt{\frac{2}{L}}\sin\frac{\pi n x}{L} \qquad Y_s(y) = \sqrt{\frac{2}{L}}\sin\frac{\pi s y}{L}$

$E_{x_n} = \frac{\pi^2\hbar^2 n^2}{2mL^2} \qquad E_{y_s} = \frac{\pi^2\hbar^2 s^2}{2mL^2}$

$n, s = 1, 2\ldots$

Total (kinetic) energy $E_{n,} = E_{xn} + E_{ys}$

$$E_{n,s} = \frac{\pi^2\hbar^2}{2mL^2}(n^2+s^2)$$

If the well is ractangular $L_x \times L_y$

$$E_{ns} = \frac{\pi^2\hbar^2}{2m}\left(\frac{n^2}{L_x^2}+\frac{s^2}{L_y^2}\right)$$

Energy spectrum for 3D case (cube)

$$E_{nst} = \frac{\pi^2\hbar^2}{2mL^2}(n^2+s^2+t^2)$$

	1D		2D		3D	
Ground state energy	$n=1$	$\frac{\pi^2\hbar^2}{2mL^2} = E_1$	$n=s=1$	$2E_1$	$n=s=t=1$	$3E_1$
		$+3E_1$		$+3E_1$		$+3E_1$
1^{st} excited state	$n=2$	$4E_1$	$n=1\ s=2$ $n=2\ s=1$	$5E_1$	$n=s=1,\ t=2$ and	$6E_1$
		$+5E_1$		$+3E_1$		$+3E_1$
2^{nd} excited state	$n=3$	$9E_1$	$n=s=2$	$8E_1$	$n=1\ s=t=2$ and	$9E_1$

While the energy levels are different in these three cases even for the ground state, we are only able to measure the difference between energy levels ~~[illegible]~~ via induced transitions

$$\hbar\omega_{ij} = E_i - E_j$$

and there is more similarities b/w the three ~~spectra~~ absorption spectra, especially for low-energy states