

Potentially useful information

Spin-1/2 particle

$$\hat{S}_z = \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \hat{S}_x = \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \hat{S}_y = \frac{\hbar}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

Eigenstates for the spin operators:

$$|+z\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \quad |-z\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}; \quad |+x\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}; \quad |-x\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}; \quad |+y\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}; \quad |-y\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

Spin-1 particle

$$\hat{S}_z = \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad \hat{S}_x = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \hat{S}_y = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}.$$

Eigenstates of the \hat{S}_z operator (in the z -basis):

$$|1, 1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}; \quad |1, 0\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}; \quad |1, -1\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

The commutator of two operators is defined as $[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$.

Potentially useful mathematical expressions

$$i \cdot i = -1; \quad i \cdot (-i) = 1; \quad 1/i = -i;$$

$$e^{i\phi} = \cos \phi + i \sin \phi; \quad e^{i\pi/2} = i; \quad \cos \phi = (e^{i\phi} + e^{-i\phi})/2; \quad \sin \phi = (e^{i\phi} - e^{-i\phi})/2i;$$

$$e^{i\pi} = -1; \quad e^{\pm i\pi/2} = \pm i; \quad e^{\pm i\pi/4} = \frac{1}{\sqrt{2}} \pm \frac{i}{\sqrt{2}};$$

$$|e^{i\phi}|^2 = 1;$$

$$\cos 2\phi = \cos^2 \phi - \sin^2 \phi; \quad \sin 2\phi = 2 \sin \phi \cos \phi$$