

Quantum Physics I (PHYS313)

Complex number practice (optional, not graded)

Since complex numbers are used in quantum calculations all the time, it will help to review the basic concepts. For many calculations we will use complex numbers in *polar form*: $z = r e^{i\phi} = r \cos(\phi) + ir \sin(\phi)$.

Practice 1 Change to polar form:

(i) $-i$, (ii) $\frac{1}{2} - \frac{\sqrt{3}i}{2}$, (iii) $1 + i$, (iv) $1 - i$, (v) $(1 + i)/(1 - i)$.

Practice 2 Find real and imaginary parts of: (i) i^5 , (ii) $(2 + 3i)/(1 + 6i)$, (iii) $e^{i\pi/6} - e^{-i\pi/6}$.

Practice 3 For $z_1 = 2e^{i\pi/4}$ and $z_2 = e^{-3i\pi/4}$, find:

(i) z_1^2 , (ii) $|z_1|^2$, (iii) $z_1 + z_2$, (iv) $z_1 \cdot z_2$, (v) z_1/z_2 .

Practice 4 This one is a little trickier. Use polar form and complex exponents to prove that $\cos 4\theta = 8 \cos^4 \theta - 8 \cos^2 \theta + 1$.

Find the expression for $\sin 4\theta$ in terms of $\sin \theta$ and $\cos \theta$?

Practice 5 Calculate:

(i) $(1 + i)^{10}$, (ii) $(1 + i)^9/(1 - i)^9$.

If you use polar form, the calculations should be one-liners.