Rough composition of the final exam for PHYS 313: Quantum Mechanics I

Problem 1: Free particle interaction with a square potential barrier or well, calculations of reflection and/or transmission probabilities.

Problem 2: Spin-1/2 or spin-1 particle in various bases, calculations of the spin measurements outcomes and their probabilities, expectation values, time evolution of spin states and their expectation values in magnetic field.

Problem 3: Hydrogen atom: energy spectrum, degeneracies, possible measurement outcomes and averages for energy, angular momentum $\hat{L}^2$ and its z-component $\hat{L}_z$ of superposition states, time evolution of quantum superpositions.

Problem 4: Fundamentals of quantum mechanics, tasks may include: wave function normalization and decomposition; calculations of eigenvalues and eigenstates; combined action of operator, presented as 2×2 or 3×3 matrices; commutation relations.

Problem 5: A particle in a 1D infinite square well, calculations of probabilities of being in a certain state, average values for energy, position or momentum.

Problem 6: Simple harmonic oscillator, wavefunctions, energy spectrum, ladder operators.

Potentially useful information

Spin-1/2 particle

$$\hat{S}_z = \frac{\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\ \hat{S}_x = \frac{\hbar}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\ \hat{S}_y = \frac{\hbar}{2}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$$

Eigenstates for the spin operators:

$$|+z\rangle = \begin{pmatrix}1\\0\end{pmatrix};\ |-z\rangle = \begin{pmatrix}0\\1\end{pmatrix};\ |+x\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix};\ |-x\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix};\ |+y\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix};\ |-y\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\end{pmatrix}$$

Spin-1 particle

$$\hat{S}_z = \hbar\begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -1\end{pmatrix}\ \hat{S}_x = \frac{\hbar}{\sqrt{2}}\begin{pmatrix}0 & 1 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}\ \hat{S}_y = \frac{\hbar}{\sqrt{2}}\begin{pmatrix}0 & -i & 0\\ i & 0 & -i\\ 0 & i & 0\end{pmatrix}.$$

Eigenstates of the $\hat{S}_z$ operator (in the z-basis):

$$|1,1\rangle = \begin{pmatrix}1\\0\\0\end{pmatrix};\ |1,0\rangle = \begin{pmatrix}0\\1\\0\end{pmatrix};\ |1,-1\rangle = \begin{pmatrix}0\\0\\1\end{pmatrix}$$

The commutator of two operators is defined as $[\hat{A},\hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$.

Kronecker delta symbol:

$\delta_{nk} = \begin{cases} 1 & n = k \\ 0 & n \neq k \end{cases}$

Differential equations:

$\dfrac{d^2y}{dx^2} = -k^2 y$, possible solutions $y_{1,2} = \sin(kx)$ and $\cos(kx)$ or $y_{1,2} = e^{\pm ikx}$

$\dfrac{d^2y}{dx^2} = \kappa^2 y$, possible solutions $y_{1,2} = e^{\pm\kappa x}$

Orthogonality of the trigonometric functions:

$\int_0^L \sin\frac{\pi nx}{L}\sin\frac{\pi kx}{L}dx = \frac{L}{2}\delta_{nk}$,

$\int_0^L \cos\frac{\pi nx}{L}\cos\frac{\pi kx}{L}dx = \frac{L}{2}\delta_{nk}$,

$\int_0^L \sin\frac{\pi nx}{L}\cos\frac{\pi kx}{L}dx = 0$

Simple Harmonic Oscillator (SHO) wave functions:

$\psi_0(x) = \sqrt[4]{\frac{m\omega}{\pi\hbar}}e^{-\frac{m\omega}{2\hbar}x^2},\ E_0 = \frac{1}{2}\hbar\omega$

$\psi_1(x) = \sqrt[4]{\frac{m\omega}{\pi\hbar}}\sqrt{\frac{2m\omega}{\hbar}}xe^{-\frac{m\omega}{2\hbar}x^2},\ E_1 = \frac{3}{2}\hbar\omega$

$\psi_2(x) = \sqrt[4]{\frac{m\omega}{4\pi\hbar}}\left(\frac{2m\omega}{\hbar}x^2 - 1\right)e^{-\frac{m\omega}{2\hbar}x^2},\ E_2 = \frac{5}{2}\hbar\omega$

SHO raising and lowering operators: $\hat{a} = \sqrt{\frac{m\omega}{2\hbar}}\hat{x}\sqrt{\frac{m\omega}{2\hbar}}\hat{x} + \frac{1}{\sqrt{2m\omega\hbar}}\hat{p};\ \hat{x} = \sqrt{\frac{\hbar}{2m\omega}}\left(\hat{a} + \hat{a}^\dagger\right);\ \ \hat{p} = -i\sqrt{\frac{m\hbar\omega}{2}}\left(\hat{a} - \hat{a}^\dagger\right)$

$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle,\ \hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$

Potentially useful mathematical expressions

$$i\cdot i = -1;\ i\cdot(-i) = 1;\ 1/i = -i;$$
$$e^{i\phi} = \cos\phi + i\sin\phi;\ \cos\phi = (e^{i\phi} + e^{-i\phi})/2;\ \sin\phi = (e^{i\phi} - e^{-i\phi})/2i;$$
$$e^{i\pi} = -1; e^{\pm i\pi/2} = \pm i; e^{\pm i\pi/4} = \frac{1}{\sqrt{2}} \pm \frac{i}{\sqrt{2}};$$
$$\left|e^{i\phi}\right|^2 = 1;$$
$$\cos 2\phi = \cos^2\phi - \sin^2\phi;\ \sin 2\phi = 2\sin\phi\cos\phi$$