Physics 213 Midterm test #2
October 21 2016

Name (please print): ____________________________

This test is administered under the rules and regulations of the honor system of the College of William & Mary.

Signature: ______________________________________

Show all work to receive credit, and circle your final answers. This exam is closed book, and you can use calculators only for simple arithmetical operations.

Problem 1 (30 points)

Consider a double-slit interference experiment in which the slit spacing is d=0.1mm, and the projection screen is located L=50cm behind the slits.

a) Assuming monochromatic illumination at normal incidence, if the observed separation between neighboring interference maxima at the center of the projection screen is Δx=2.5mm, what is the wavelength of the light illuminating the screen?

b) A thin diamond film is placed in front of one of the slits, such that it shifts the phase of the light by half a wavelength. What will change in the interference picture? By what distance the interference maxima will shift on the screen?

c) What is the minimum thickness of the film to accomplish the required half wave-length retardation, considering the diamond index of refraction of n_d=2.63.

a) The angular difference b/w two consecutive max $\Delta\theta = \lambda/d$. Thus the separation on the screen $\Delta x = L\Delta\theta$

$$\Delta x = L\,\lambda/d \quad \Rightarrow \quad \lambda = \frac{\Delta x}{L}\cdot d = 500nm$$

b) With the extra phase shift the conditions for constructive and destructive interference are reversed, so the bright stripe is replaced with dark and vice versa. That is equivalent to all stripes shifting by $\Delta x/2 = 1.25mm$

c) $\lambda_d = \frac{\lambda_0}{n}$ $\qquad \lambda_0 = 500nm$

Difference in the optical path in air and in diamond

$$-k_{air}\cdot d + k_{diamond}\cdot d = \cancel{\pi/2}\ \pi$$

$$-\frac{2\pi d}{\lambda_0} + \frac{2\pi n_d\cdot d}{\lambda_0} = \pi \quad \Rightarrow \quad d = \frac{\lambda_0}{2(n-1)} = 153nm$$

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Problem 2 (30 points)

a) Consider a diffraction grating having 5000 lines per centimeter. Find the angular locations of the principal maxima when the grating is illuminated at normal incidence by (i) red light of wavelength 700*nm*, and (ii) violet light of wavelength 400*nm*.

b) Suppose that instead of the monochromatic light one uses a sodium discharge lamp that produces light at two closely spaced wavelengths of 589.0*nm* and 589.6*nm*. Will you be able to resolve these two doublet in the first or second diffraction order?

c) Suppose further that the experiment is submerged into a jar of transparent glycerin (n_g=1.6). How would the diffraction picture change? Would it be easier to resolve the optical doublet under these new conditions?

a) Principle maxima $\theta_{max} \approx \frac{m\lambda}{d}$ (assuming $\theta_{max} \approx \sin\theta_{max}$ for $\theta_{max} \ll 1$)

$d = \frac{10^{-2} m}{5000} = 2 \cdot 10^{-6} m$

$\theta_{max} = m \cdot \frac{\lambda}{d}$ $m = 0, 1, 2 \ldots$

$\theta^{red}_{max} = m \cdot \frac{7 \cdot 10^{-7} m}{2 \cdot 10^{-6} m} \approx 0.35 m$ (0.35 rad, 0.7 rad, 1.05 rad...)

$\theta^{violet}_{max} = m \cdot \frac{4 \cdot 10^{-7} m}{2 \cdot 10^{-6} m} \approx 0.2 \cdot m$ (0.2 rad, 0.4 rad, 0.6 rad...)

b) For two closely-spaced lines.

$\Delta\theta = \theta(\lambda_1) - \theta(\lambda_2) = \frac{\Delta\lambda}{d}$ (first order)

$= \frac{2\Delta\lambda}{d}$ (second order)

Sharpness of the line is determined by the width of diffraction peak

$\frac{\pi N d \sin\theta}{\lambda} = \pi$

$\sin\Delta\theta \approx \Delta\theta = \frac{\lambda}{N d} \approx 3 \cdot 10^{-4} rad$

for $N \approx 1000$ lines

$\Delta\theta = (2\times) \frac{6 \cdot 10^{-10} m}{2 \cdot 10^{-6} m} = (2\times)\ 3 \cdot 10^{-4} rad$

for the second order

So it will be hard to resolve them

c) Higher index of refraction means smaller wavelength, and all the maxima shift toward the center and thus get closer. It will make it harder to resolve the doublet.

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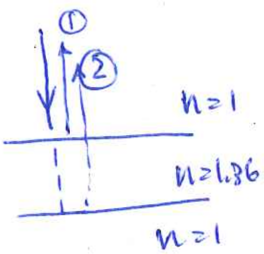
Problem 3 (30 points)

a) A soap bubble d=250nm thick is illuminated by white light. The index of refraction of the soap film is n_s=1.36. Which colors are not seen in the reflected light? Which colors appear bright in the reflected light?

b) Suppose that the identical soap film is made on the surface of a polished glass (n_g=1.5). Will this affect the visible coloration of the film? If yes, answer the questions from part (a) for such film.

a) Thin film interference



① top reflection : extra π phase shift

② bottom reflection: $2kd = \frac{4\pi}{\lambda} \cdot d$ phase shift

Constructive interference: $\Delta\varphi = 0, 2\pi, 4\pi, \ldots$

$$\frac{4\pi d n}{\lambda} - \pi = 2\pi \cdot m$$

$$2dn = \lambda(m + 1/2) \Rightarrow \lambda_{bright} = \frac{2dn}{m + 1/2}$$

m=0 λ_{bright} = 1360nm (IR, invisible)

m=1 λ_{bright} = 453nm (blue ~~green~~)

m=2 λ_{bright} = 272nm (ultraviolet, invisible)

Destructive interference $\Delta\varphi = \pi, 3\pi \ldots$

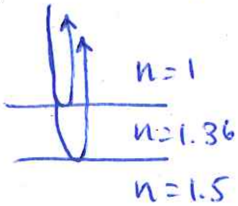
$$\frac{4\pi d n}{\lambda} - \pi = 2\pi(m - 1/2)$$

$$2nd = \lambda m \Rightarrow \lambda_{dark} = \frac{2dn}{m} = 680\,nm$$

red

The film will look blue, and red tones will be missing

b) Thin soap film on glass



top reflection: extra π phase shift

bottom reflection: extra π phase shift + $2kd$

$\Delta\varphi = \frac{4\pi d n}{\lambda}$ → conditions for constructive and destructive interference switch, film would appear red-colored

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Problem 4 (10 points)

Estimate how large the lens of a camera carried by an artificial satellite orbiting the Earth at an attitude of 150 miles would have to be in order to resolve features on the Earth's surface a foot in diameter.

$$\Delta\theta = \frac{1.22\lambda}{D}$$ where D is the lens diameter

To resolve features of size $d = 1$ ft ≈ 30 cm $= 0.3$ m
at the distance $L = 150$ miles ≈ 240 km $= 2.4 \cdot 10^5$ m

$$\Delta\theta \leq \cancel{0.3m}\ \frac{d}{L} = \frac{0.3m}{2.4 \cdot 10^5 m} = 1.25 \cdot 10^{-6}$$

$$\cancel{D \sim \left(\frac{\Delta\theta}{1.22\lambda}\right) = \left(\frac{1.25 \cdot 10^{-6}}{1.22 \cdot 5 \cdot 10^{-7} m}\right)}$$

$$D \sim \frac{1.22\lambda}{\Delta\theta} = \frac{1.22 \cdot 5 \cdot 10^{-7} m}{1.25 \cdot 10^{-6}} \approx 0.5 m$$

This is an order-of-magnitude estimate, since the wavelength of light changes in the range $400 \div 700$ nm

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