

Ray optics vs Wave optics

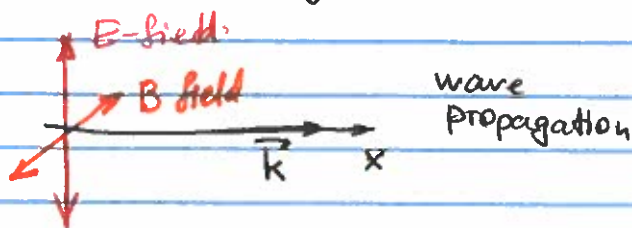
Characteristics of a ray:

- propagation direction
- position (crossing point)
- color (for artistic purposes)
- power (energy deposited / time) (brightness)

We can think of a ray as a laser beam
 & Plane wave describes an ideal laser beam (if we ignore the transverse shape)

Propagation direction $\rightarrow$ wave vector $\vec{k}$
 Color $\rightarrow$ frequency (or wavelength) ω
 Brightness $\rightarrow$ amplitude E_0

Electro magnetic wave: oscillating electric and magnetic field



Normally $\vec{E}$, $\vec{B}$ and $\vec{k}$ are perpendicular to each other

$$\vec{E} = \vec{E}_0 \cos(kx - \omega t)$$

$\vec{k}$ is in x-direction
 in general $\vec{k} \cdot \vec{r}$

~~Recall~~ frequency $\omega = \frac{2\pi}{T}$ rate of oscillations in time

wavelength in vacuum

$$\lambda = \frac{2\pi}{k} = \frac{2\pi c}{\omega}$$

length of one oscillation

$$c = 3 \cdot 10^8 \text{ m/s}$$

speed of light in vacuum

wavelength
in a material

$$v_{\text{mat}} = \frac{c}{n_{\text{mat}}}$$

$$\lambda_{\text{mat}} = \frac{2\pi v_{\text{mat}}}{\omega} = \frac{2\pi c}{\omega n_{\text{mat}}} = \frac{\lambda_{\text{vac}}}{n_{\text{mat}}}$$

Frequency of light does not change when it crosses material boundaries, but wavelength does.

Typically we specify the wavelength in vacuum

Visible light: $\lambda = 400 - 700 \text{ nm} = 4 \cdot 10^{-7} - 7 \cdot 10^{-7} \text{ m}$

$$\text{frequency} = \frac{2\pi c}{\lambda} = 2.7 \cdot 10^{15} \frac{\text{rad}}{\text{s}} - 4.7 \cdot 10^{15} \frac{\text{rad}}{\text{s}}$$

$$4.3 \cdot 10^{14} \text{ Hz} - 7.5 \cdot 10^{14} \text{ Hz}$$

~~Sometimes~~ In spectroscopy people often use wave number - inverse wavelength

$$\text{wave number} = \frac{1}{\lambda} = \frac{k}{2\pi} \quad \text{measured in } 1/\text{cm}$$

For visible light

$$\text{wave number} = 14300 1/\text{cm} - 25000 1/\text{cm}$$

Magnetic component:

$$\vec{E} = \vec{E}_0 \cos(kx - \omega t)$$

$$\vec{B} = \vec{B}_0 \cos(kx - \omega t)$$

(perpendicular to each other
in y-z plane)

$$|\vec{B}_0| = \frac{|\vec{E}_0|}{c} \text{ in vacuum} \quad \text{or} \quad = \frac{|\vec{E}_0|}{v_{\text{mat}}} \text{ in material}$$

Brightness of a ray = how much energy it carries

$$u = \frac{\text{Energy density}}{\text{unit volume}} = \frac{1}{2} \epsilon_0 E^2 + \frac{1}{2} \frac{1}{\mu_0} B^2$$

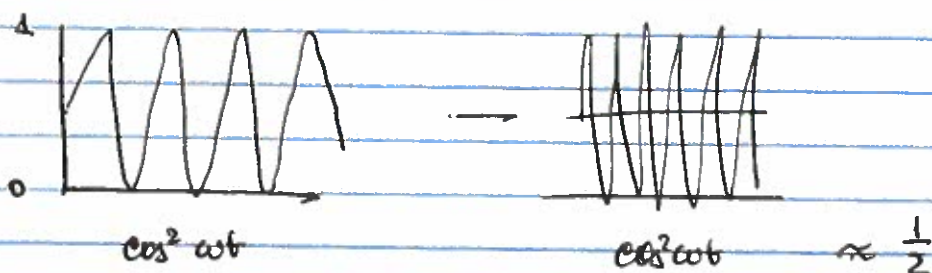
$\epsilon_0 \mu_0 = \frac{1}{c^2}$
↑ susceptibility of vacuum
↑ permeability of vacuum

need to match units

Since E and B oscillate in time, energy oscillate in time as well

$$u(x,t) = \left(\frac{1}{2} \epsilon_0 E_0^2 + \frac{1}{2} \frac{1}{\mu_0} B_0^2 \right) \cos^2(kx - \omega t)$$

However, ω is very high - too high for most things to notice! instead, we see an average

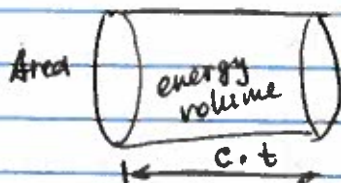


$$\text{Average energy density} = \frac{1}{2} \left[\frac{1}{2} \epsilon_0 E_0^2 + \frac{1}{2} \frac{1}{\mu_0} B_0^2 \right] \approx \frac{1}{2} \epsilon_0 E_0^2$$

$$(u_{\text{ave}} = \frac{1}{2} \epsilon_0 E_0^2)$$

Energy ~~density~~ flow rate = ~~energy~~ ~~flow~~

$$\text{Laser intensity } I = \frac{\text{energy}}{\text{area} \cdot \text{time}} = c \cdot u_{\text{ave}} = \frac{\epsilon_0 c}{2} E_0^2$$



More generally, the energy flow can be calculated using a Poynting vector

$$\vec{N} = \frac{1}{\mu_0} \underbrace{\vec{E} \times \vec{B}}_{\text{cross product}}$$

since $\vec{N}$ is perpendicular to $\vec{E}$ and $\vec{B}$, it is collinear to $\vec{E}$, along beam propagation

Averaged over time $|\vec{N}|_{\text{ave}} = \frac{1}{2} E_0^2 \underbrace{\left\{ \frac{\epsilon_0}{\mu_0} \right\}}_{Z_0 = 377 \Omega} = \frac{1}{2} \epsilon_0 c E_0^2 = I_{\text{ave}}$
Impedance of ~~vacuum~~ free space

Average in time

~~How much~~ momentum density

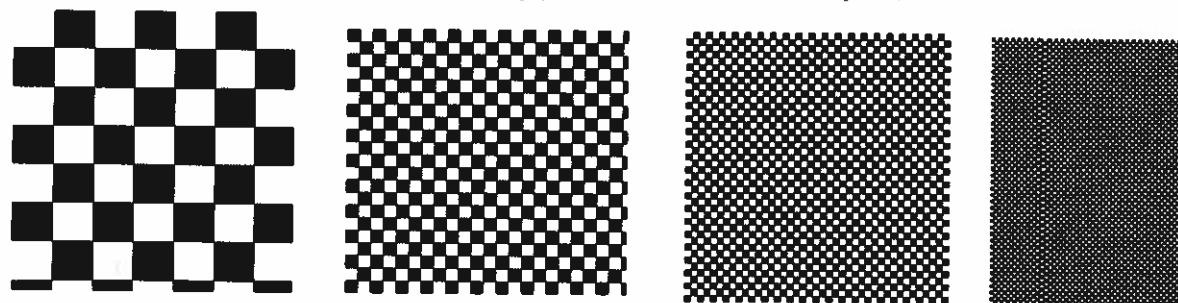
$$|\vec{p}|_{\text{ave}} = \frac{|\vec{N}|_{\text{ave}}}{c^2} = \frac{\epsilon_0}{2c} E_0^2$$

light wave carries energy and momentum, and produces light pressure

$$\text{Radiation pressure} = \frac{I_{\text{ave}}}{c}$$

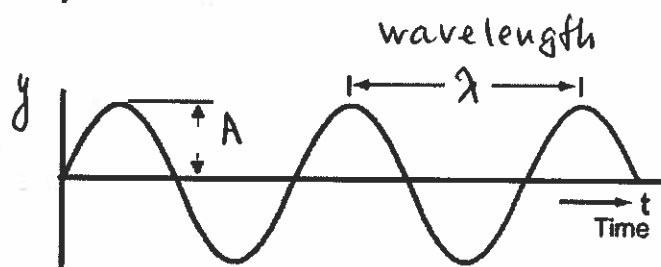
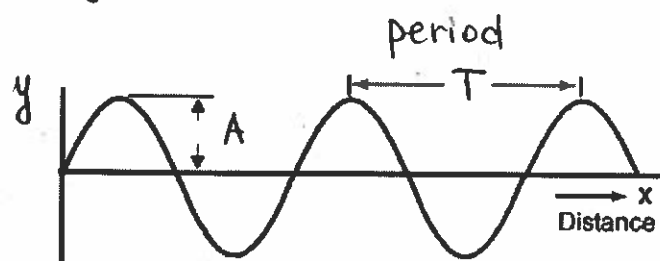
Q Google "Solar sail"

Illustration of temporal/spatial averaging of wave oscillations



Reminder of wave description terms

Wave: $y(x,t) = A \cos(kx - \omega t) = A \cos\left(\frac{2\pi}{\lambda}x - \frac{2\pi}{T}t\right)$
(generic)



A - amplitude

Angular frequency
 $\omega = \frac{2\pi}{T}$ [in $\frac{\text{rad}}{\text{s}}$]

Frequency
 $f = \frac{\omega}{2\pi} = \frac{1}{T}$ [in Hz]

Wave vector

$$k = \frac{2\pi}{\lambda}$$