

Two-wave interference

We assume the addition of two waves of same frequency and polarization. And for simplicity let's make them same amplitude as well.

Wave 1: $\vec{E}_1(x,t) = E_0 \cos(kx - \omega t)$

Wave 2: $\vec{E}_2(x,t) = E_0 \cos(kx - \omega t + \varphi)$
or $= E_0 \cos(k(x + \Delta x) - \omega t)$] $\varphi = k\Delta x = \frac{2\pi\Delta x}{\lambda}$

φ - phase delay Δx - extra distance traveled

Constructive interference: two waves are oscillating synchronously

$$\vec{E}_{\text{tot}} = 2E_0 \cos(kx - \omega t)$$

$$I_{\text{tot}} = 4I_0$$

where $I_0 = \frac{c\epsilon_0}{2} E_0^2$ intensity of a single wave

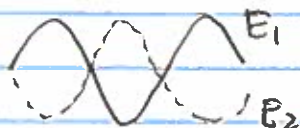
When constructive interference happens!

$$\varphi = 0, 2\pi, 4\pi, \dots \quad m \cdot 2\pi \quad m = 0, 1, 2, \dots$$

$$\text{or } \frac{2\pi\Delta x}{\lambda} = 2\pi \cdot m \quad \Delta x = m\lambda$$

So if two waves originated from the same source, but one traveled integer # of wavelengths extra, they interfere constructively

Destructive interference! Two waves are oscillating opposite of each other



$$E_{\text{tot}} = E_1 + E_2 = 0$$

When it happens:

$$\text{Since } \cos(d + \pi) = -\cos d$$

the waves will be opposite if

$$\varphi = \pi, 3\pi, 5\pi \dots \pi + 2\pi n \quad n=0,1,\dots$$

or

$$\frac{2\pi\Delta x}{\lambda} = \pi + 2\pi n \Rightarrow \frac{\Delta x}{\lambda} = \frac{1}{2} + n \Rightarrow \Delta x = \lambda/2 + \lambda n$$

The two waves originated from the same source will interfere destructively (no light) if one of them ~~traveled~~ ^{traveled} semi-integer number of wavelengths

$$\text{General case: } \vec{E}(x,t) = E_0 \cos(kx - \omega t) + E_0 \cos(kx - \omega t + \varphi)$$

$$= 2E_0 \cos\left(\frac{\varphi}{2}\right) \cos(kx - \omega t + \varphi/2) = 2E_0 \cos\left(\frac{\pi\Delta x}{\lambda}\right) \cos(kx - \omega t + \varphi/2)$$

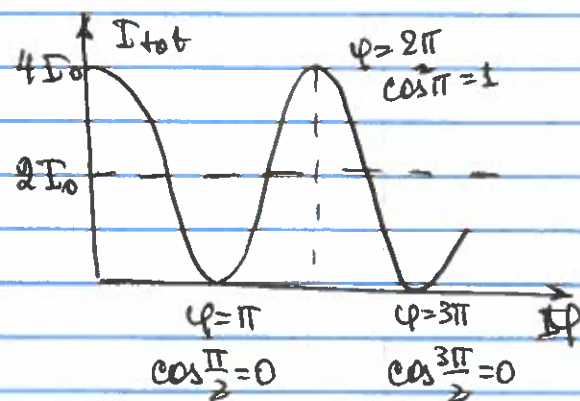
E_{tot}

$$\text{Total intensity } I_{\text{tot}} = \frac{c\epsilon_0}{2} (E_{\text{tot}})^2 = 4\cos^2\left(\frac{\varphi}{2}\right) \left[\frac{c\epsilon_0}{2} E_0^2\right]$$

$$I_{\text{tot}} = 4\cos^2\left(\frac{\varphi}{2}\right) \cdot I_0$$

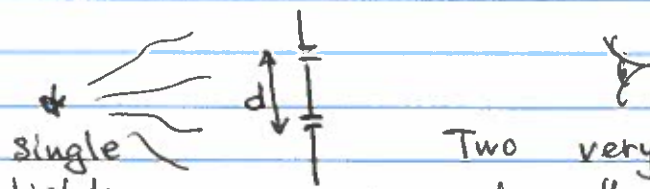
If no interference

$$I_{\text{no int}} = 2 \cdot I_0$$



So how can we make two identical waves and then bring them together?

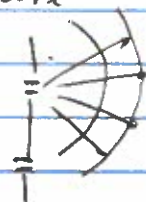
Two-slit interference



single light source

Two very small holes near each other, each one is illuminated the same way, and they "create" two identical ^{point} light sources.

The interference pattern we see will depend on how far we are from the sources. Distances comparable with the distance b/w the slits — near field. In principle, it will give us more information, but much more complex



wave front — the surface perpendicular to all rays, have constant phase.

[interlude]

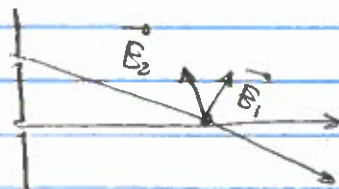
Electromagnetic wave from a point source

$$\vec{E}(r, t) = \left[E_0 \frac{1}{kr} \right] \cos(kr - \omega t)$$

amplitude

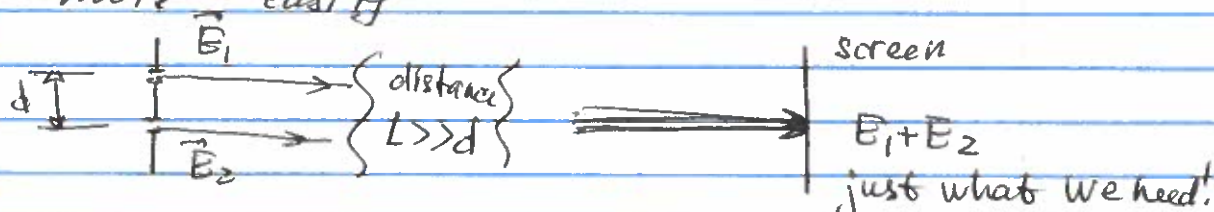
The amplitude of the wave becomes smaller and smaller as we move further from the source, since the original energy is dispersed across larger and larger distance

Two slits near field interference

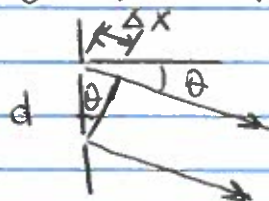


Need to calculate vector sum of two waves ... messy!

Far field - distances are much larger than the distance b/w two slits. Then the two waves are basically identical in the direction of propagation and polarization, and can be added much more easily



So what is the phase difference? to see interference



If two waves "depart" in the direction of angle θ , one of them will travel extra

$$\Delta x = d \sin \theta \approx d \cdot \theta \text{ distance (for small } \theta)$$

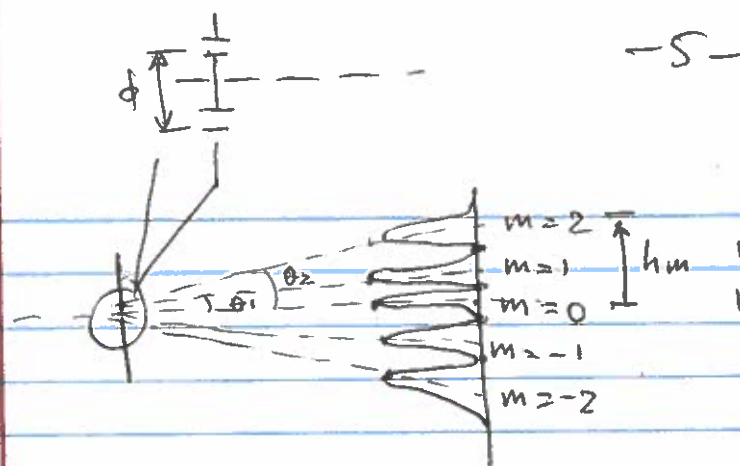
That difference will determine how these waves interfere

Constructive interference: $\Delta x_{\max} = d \cdot \theta_{\max} = m\lambda$

$$\theta_{\max}^{(m)} = m \frac{\lambda}{d}$$

Destructive interference: $\Delta x_{\min} = \frac{\lambda}{2} + m\lambda = d \theta_{\min}$

$$\theta_{\min}^{(m)} = (m + \frac{1}{2}) \frac{\lambda}{d}$$



Distance to the
 m -th interference
 maximum is h_m
 $h_m = L \cdot \tan \theta_{\max}^{(m)}$
 $\propto L \cdot \theta_{\max}^{(m)} = m \frac{L \cdot \lambda}{d}$

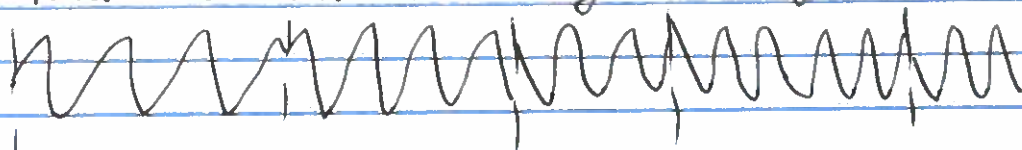
m - the order of the
 interference maxima.

The farther two slits are from each other,
 the ~~harder~~ closer are the maxima to
 each other, and even under ideal
 Since any imperfections wash out or
 distort the perfect lines, ~~the~~ realistically
 the far-separated interference fringes
 are ~~harder~~ getting hard to resolve

Why it is very hard to see interference
 without using a laser?

1. Different colors have different angles
 (or fringe positions), since $\theta_{\max} \propto \lambda$

2. Any light wave is not a perfect consistent
 oscillation, ~~it~~ even for a monochromatic
 wave it will maintain a phase for some
 time, then it will randomly change



The average duration for which a wave has
 defined phase is ~~also~~ called its
 coherence length

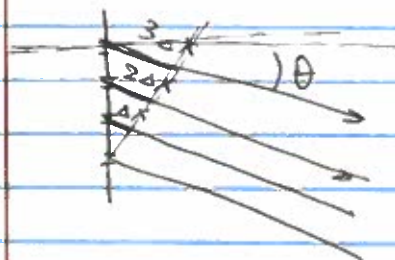
coherence length is usually very small fraction

Diffraction grating (should be called "interference")

Many identical slits separated by

grating

same distance d



As before, we consider far field, so all phase difference comes from the original offset

$$\Delta x = d \cdot \theta \text{ for the first, } 2\Delta x$$

for the second, ... $N\Delta x$ for the last

$$E_{\text{tot}} = E_0 \cos(kx - \omega t) + E_0 \cos(kx - \omega t + k\Delta x) + E_0 \cos(kx - \omega t + 2k\Delta x) + \dots + E_0 \cos(kx - \omega t + N \cdot k\Delta x)$$

Constructive interference - all waves are in sync

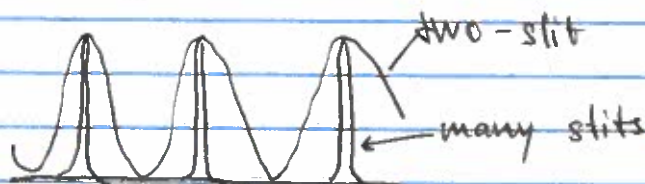
$$\Delta x = m \lambda = d \cdot \theta_{\text{max}}^{(m)}$$

$$\theta_{\text{max}}^{(m)} = m \frac{\lambda}{d} \quad (\text{just as for two slits})$$

But! Total amplitude $E_{\text{tot}} = N \cdot E_0$

$$I_{\text{tot}} \propto |E_{\text{tot}}|^2 = N^2 I_0 \quad (\text{vs } N \cdot I_0 \text{ for non-interfering light})$$

Very bright (and sharp!) maxima



Interference is a distinct feature of the quantum realm

Wave-particle duality -- any particle can behave as a wave packet with the wavelength $\lambda = \frac{2\pi\hbar}{p}$ (de Broglie wavelength)

$$p = mv \quad (\text{non-relativistic case, } v \ll c)$$

$$\hbar = 10^{-34} \text{ J}\cdot\text{s} \quad \text{Planck's constant}$$

Because $\hbar$ is so ridiculously small, λ is too small to notice for most normal-sized objects

An electron with energy 100 eV
 $\lambda \sim 1.2 \cdot 10^{-10} \text{ nm}$ (a size of an atom)
 $m_e = 9.10^{-31} \text{ kg}$
 $v_e = 6 \cdot 10^6 \text{ m/s}$ (2% of speed of light)

If we get very slow electrons
 $v_e = 1000 \text{ m/s}$ $\lambda = 7 \cdot 10^{-7} \text{ meters}$
 (comparable with the visible wavelength)