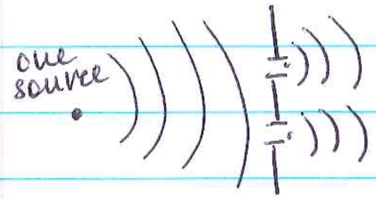
Examples of interference

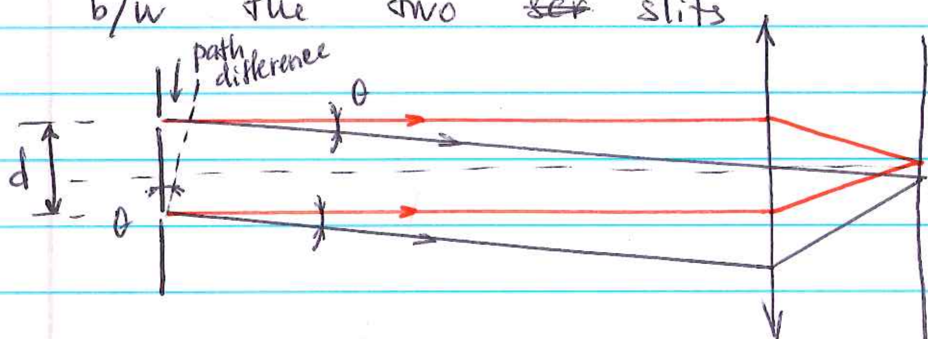
To ensure the <u>coherence</u> of multiple light sources, majority of the experiments uses wave front division

1. Double - slit experiment



A screen with two identical slits is placed in front of a light source, and each slit starts acting as an independent light source, but the two have identical ~~phases~~ frequencies, and are in phase

Far- field geometry: the observation occures much farther than the distance b/w the two ~~ser~~ slits



Two paths are identical constructive interference

Two beams travel different distance, and thus arrive with some phase different

The path difference $\Delta z = d \sin\theta$

Phase difference $k\Delta z = \frac{2\pi}{\lambda}\Delta z = 2\pi \frac{d}{\lambda} \sin\theta$

Constructive interference

$$\frac{2\pi d}{\lambda}\sin\theta = 2\pi \cdot m \qquad m = 0, \pm 1, \pm 2 \ldots$$

$$d\sin\theta_{max} = m \cdot \lambda$$

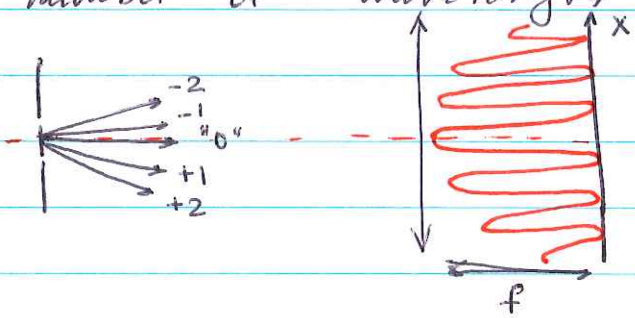
Path difference is equal to the integer number of wavelengths.

Destructive interference

$$\frac{2\pi d}{\lambda}\sin\theta = 2\pi m + \pi = 2\pi(m + 1/2)$$

$$d\sin\theta = \lambda(m + 1/2)$$

Path difference is equal to the semi-integer number of wavelength



$$x = f\sin\theta$$

$$x_{max} = f\sin\theta_{max} = f \cdot \frac{m\lambda}{d}$$

$$x_{min} = f\sin\theta_{min} = f \cdot \frac{(m+1/2)\lambda}{d}$$

For two identical slits $I_{max} = 4I_0$ (when I_0 is the max intensity from the single slit) and $I_{min} = 0$

Visibility $V = \dfrac{I_{max} - I_{min}}{I_{max} + I_{min}}$

For the "ideal" interference $V = 1$

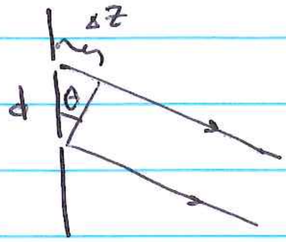
No interference $V = 0$

The closer the visibility is to 1, the better matched are the properties of the two slits.

Diffraction grating:

Many identical slits!

$N = 2$



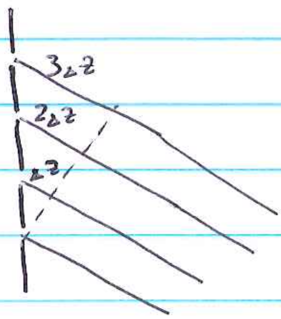
$$E_{total} = E_0 \cos(kz - \omega t) + E_0 \cos(kz - \omega t + \Delta\varphi)$$

$$= 2E_0 \cos\left(kz - \omega t + \frac{\Delta\varphi}{2}\right) \cdot \cos\frac{\Delta\varphi}{2}$$

$$I(\theta) = 2E_0^2 \cos^2\frac{\Delta\varphi}{2} =$$

$$\Delta\varphi = k\Delta z = 2\pi\frac{d\sin\theta}{\lambda} \qquad = 2E_0^2 \cos^2\left(\pi\frac{d\sin\theta}{\lambda}\right)$$

Many slits:



$$E_{total} = E_0 \cos(kz - \omega t) + E_0 \cos(kz - \omega t + \Delta\varphi) + E_0 \cos(kz - \omega t + 2\Delta\varphi) + \ldots + E_0 \cos(kz - \omega t + (N-1)\Delta\varphi)$$

Math trick: $\cos\alpha = \mathrm{Re}\,(e^{i\alpha})$

$$E_{total} = \mathrm{Re}\left\{E_0 e^{i(kz-\omega t)} + E_0 e^{i(kz-\omega t+\Delta\varphi)} + \ldots + E_0 e^{i(kz-\omega t+(N-1)\Delta\varphi)}\right\}$$

$$= \mathrm{Re}\left\{E_0 e^{i(kz-\omega t)} \underbrace{\left(1 + e^{i\Delta\varphi} + e^{2i\Delta\varphi} + \ldots + e^{i(N-1)\Delta\varphi}\right)}_{\text{geometrical progression}}\right\} =$$

$$= \mathrm{Re}\left\{E_0 e^{i(kz-\omega t)} \frac{e^{iN\Delta\varphi} - 1}{e^{i\Delta\varphi} - 1}\right\} =$$

$$= \mathrm{Re}\left\{E_0 e^{i(kz-\omega t)} \frac{e^{iN\Delta\varphi/2}}{e^{i\Delta\varphi/2}} \frac{e^{iN\Delta\varphi/2} - e^{-iN\Delta\varphi/2}}{e^{i\Delta\varphi/2} - e^{-i\Delta\varphi/2}}\right\}$$

Reminder: $\frac{e^{i\alpha} - e^{-i\alpha}}{2i} = \sin\alpha$

$$= \mathrm{Re}\left\{E_0 e^{i\left(kz-\omega t + \frac{(N-1)\Delta\varphi}{2}\right)} \frac{\sin\frac{N\Delta\varphi}{2}}{\sin\frac{\Delta\varphi}{2}}\right\} =$$

$$= E_0 \cos\left(kz - \omega t + \frac{(N-1)\Delta\varphi}{2}\right) \frac{\sin N\Delta\varphi/2}{\sin\Delta\varphi/2}$$

Total intensity $I_{tot} = \langle |E_{tot}|^2 \rangle_{\text{time average}}$

$$I_{total} = \frac{E_0^2}{2} \left(\frac{\sin N \frac{\pi d \sin\theta}{\lambda}}{\sin \frac{\pi d \sin\theta}{\lambda}} \right)^2$$

What is the biggest value of $\left(\frac{\sin Nx}{\sin x} \right)^2$ - ?

Answer - N^2! $\lim_{x \to 0} \left(\frac{\sin Nx}{\sin x} \right)^2 \approx \lim \left(\frac{Nx}{x} \right)^2 = N^2$

Very strong maxima $I_{total} \approx N^2 \cdot I_0$ ← single slit

Conditions for these maxima

$\sin \frac{\pi d \sin\theta}{\lambda} = 0$ $\frac{\pi d \sin\theta}{\lambda} = \pi \cdot m$ $m = 0, \pm 1, \pm 2 \ldots$

$$\boxed{d \sin\theta_{max} = m \cdot \lambda}$$

Same as for two-slit interference

However, for the grating the peaks are much sharper $N \cdot d \cdot \Delta\theta_{max} \sim \lambda$ $\Delta\theta_{max} \sim \lambda / Nd$

Magic of quantum mechanics?

Everything can interfere

In quantum mechanics any particle can be thought as a wave

$$\lambda = \frac{2\pi\hbar}{p} = 2\pi\hbar / mv \qquad \hbar = 10^{-34}\ J\cdot s$$

p ← momentum   Plank's constant

In order to achieve any sizable wavelength the velocity must be very low

Interference of ultracold atoms, electrons, and even molecules was observed!