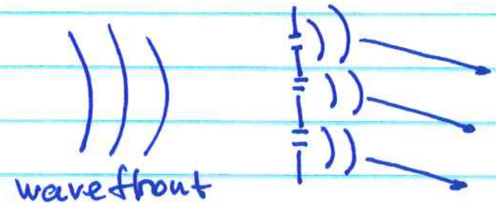
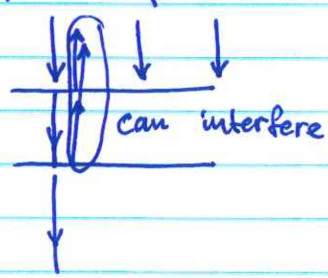
Interference by amplitude division

a) Two-slit interference, diffraction grating



wavefront division: creation of secondary 'sources' that are in phase and can interfere

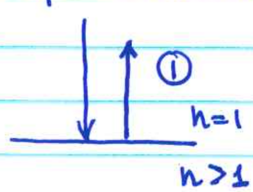
b) Plane-parallel plate



Two interfering light waves are produced by the partially reflected beams

Let's carefully monitor phase variation in two reflected beams

1. Top reflection



Incident: $E_i = E_0 \cos(kz - \omega t)$

Reflected ①: $E_{r_1} = r_1 E_0 \cos(kz - \omega t \underline{\underline{+\pi}})$

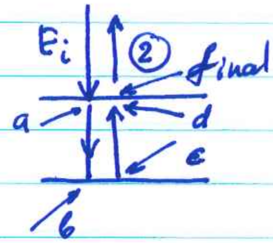
if the refractive index of the reflecting material is higher than the one the light has traveled in

air-glass — extra phase upon reflection

water-glass — —— " ——
(n=1.33) (n=1.5)

water-air — no extra phase

2.



a: $E_{2a} = t_1 E_0 \cos(k_n z - \omega t)$

$k_n = 2\pi/\lambda_n = 2\pi n/\lambda$

b: $E_{2b} = t_1 E_0 \cos(k_n z - \omega t + \underbrace{k_n d}_{\text{extra phase } \frac{2\pi n d}{\lambda}})$

c: $E_{2c} = t_1 r_2 E_0 \cos(k_n z - \omega t + k_n d)$
(assuming it is lower refractive index media outside)

d) $E_{2d} = t_1 r_2 E_0 \cos(k_n z - \omega t + 2k_n d)$

$$E_{r2} = \underbrace{t_1^2 r_2 E_0}_{\text{amplitude of the second wave}} \cos(kz - \omega t \underbrace{+ 2k_n d}_{\text{extra phase of the second wave}})$$

Phase difference: ① $+\pi$ $(+\lambda/2)$
(air- glass- air interfaces) ② $+ 2 \cdot \frac{2\pi n d}{\lambda}$
water

$$\Delta\varphi = \frac{4\pi n d}{\lambda} - \pi \overset{?}{=} \begin{cases} 2\pi m & \text{constructive} \\ 2\pi(m+\frac{1}{2}) & \text{destructive} \end{cases}$$

Constructive interference

$2 \cancel{4}\pi n d/\lambda = \cancel{2}\pi(m+1/2)$

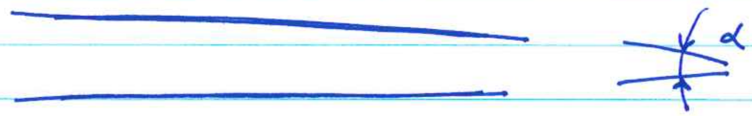
$2nd = (m+1/2)\lambda$

Destructive interference

$2 \cancel{4}\pi n d/\lambda = \cancel{2}\pi m \Rightarrow 2nd = \lambda m$

Because of the extra phase shift upon reflection, the conditions for the constructive and destructive interference has shifted!

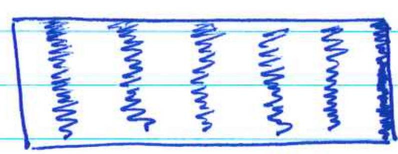
How does a wedge looks in monochromatic light?



For a wedge with a small angle α, the thickness is ~~prop~~ changing with distance

$$d(x) = d_0 - x\cdot\tan\alpha \approx d_0 - x\cdot\alpha$$

If we look up at this wedge, what we see?
the sequence of bright and dark lines



distance b/w two consequtive dark or bright lines: $2n(d_1 - d_2) = \lambda$ $\quad \Delta d = \lambda/2n$

$$\Delta x\cdot\tan\alpha = \lambda/2n$$

$$\Delta x = \lambda/(2n\cdot\tan\alpha)$$

For a small angle, Δx can be relatively large

What the very tip of the wedge is?
It is dark! $2d\cdot n \to 0$ (vanishing thickness)
destructive interference!

~~Very thin film / $d \ll \lambda$ + destructive interference~~
~~Such film would appear black!~~

What if a thing film is illuminated by a white light?

Let's assume that r_1, r_2 are small (a few percent), and $t_1, t_2 \approx 1$, so the intensities of the two reflected beams are similar

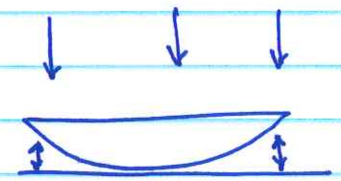
1. Very thin film: $nd \ll \lambda$ for all colors, the film appears black

2. Slightly thicker film: $2nd_e = \lambda_{blue}/2$ + constructive interference for blue but $2nd_e = \lambda_{red}/4$ - rather far from constructive for the red; the film will appear blue

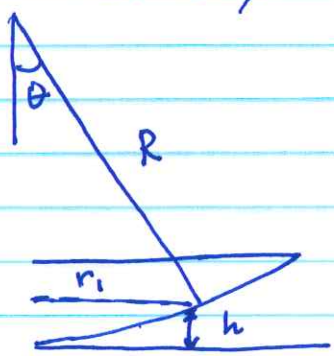
3. As thickness increases, the film will appear different spectral colors, but for slightly thicker film we can see non-spectral colors, as magenta, due to both red and blue reflecting, and mixing in our eyes.

Newton rings

How to measure a curvature of a lens if this curvature is large?



Very center is dark (destructive interference) (or since the lens and a glass are in contact, the light does not "see" be boundary b/w two glass materials)



$2h_1 = \lambda/2$ – constructive interference (first bright ring)

$2h_2 = 3\lambda/2$ – second bright ring

If r_1 is the radius of the first ring

$r_1 = R\sin\theta$

$h_1 = R(1-\cos\theta)$

For $\theta \ll 1$ $\quad r_1 \approx R\cdot\theta$

$h_1 \approx R(1-(1-\theta^2/2)) \approx R\,\theta^2/2$

$$h_1 = \frac{r_1^2}{2R}$$

Bright rings $\quad 2h = (m+\frac{1}{2})\lambda$

$$\frac{r_1^2}{R} = (m+1/2)\lambda$$

$$r_1 = \sqrt{(m+1/2)\lambda R}$$

The distrance b/w consecutive rings decreases as m grows