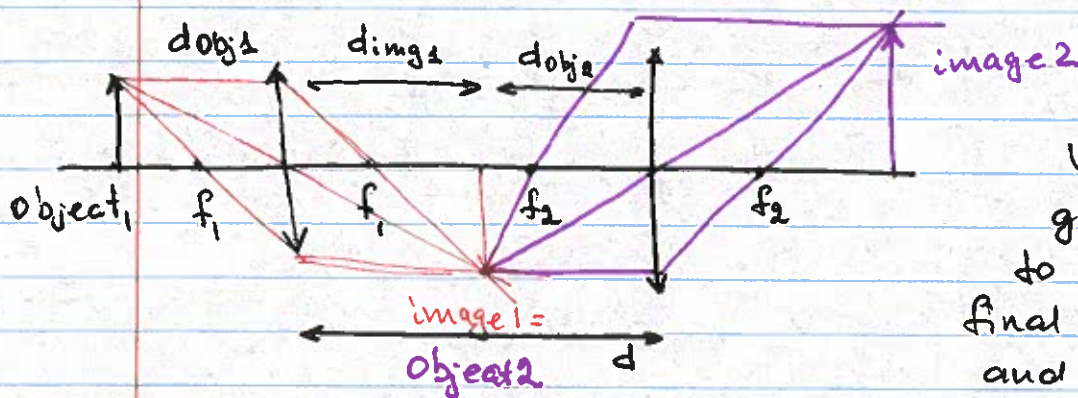


System of multiple lenses

General principle is straight forward: the image from the first lens becomes the object for the second



We can use graphical method to estimate final image location and characteristics

Exact calculations using lens formula

Lens 1 $\frac{1}{d_{obj1}} + \frac{1}{d_{img1}} = \frac{1}{f_1} \Rightarrow d_{img1} = \frac{d_{obj1} \cdot f_1}{d_{obj1} - f_1}$

$$d_{obj2} = d - d_{img1}$$

Lens 2 $\frac{1}{d_{obj2}} + \frac{1}{d_{img2}} = \frac{1}{f_2}$

$$d_{img2} = \frac{d_{obj2} \cdot f_2}{d_{obj2} - f_2} = \frac{f_2 [d(d_{obj1} - f_1) - f_1 d_{obj1}]}{(d - f_2)(d_{obj1} - f_1) - f_1 d_{obj1}}$$

if $f_1 = 5\text{cm}$ $f_2 = 7\text{cm}$ $d_{obj1} = 9\text{cm}$ and $d = 25\text{cm}$

$$d_{img1} = 11.25\text{cm} \quad d_{obj2} = 25\text{cm} - 11.25\text{cm} = 13.75\text{cm}$$

$$d_{img2} = 14.26\text{cm}$$

Magnification: $h_{img1} = m_1 \cdot h_{obj1}$ total magnification

h_{obj2} ; $h_{img2} = m_2 \cdot h_{obj2} = \boxed{m_1 \cdot m_2} h_{obj1}$

$$m_1 = - \frac{d_{\text{img1}}}{d_{\text{obj1}}}$$

$$m_2 = - \frac{d_{\text{img2}}}{d_{\text{obj2}}}$$

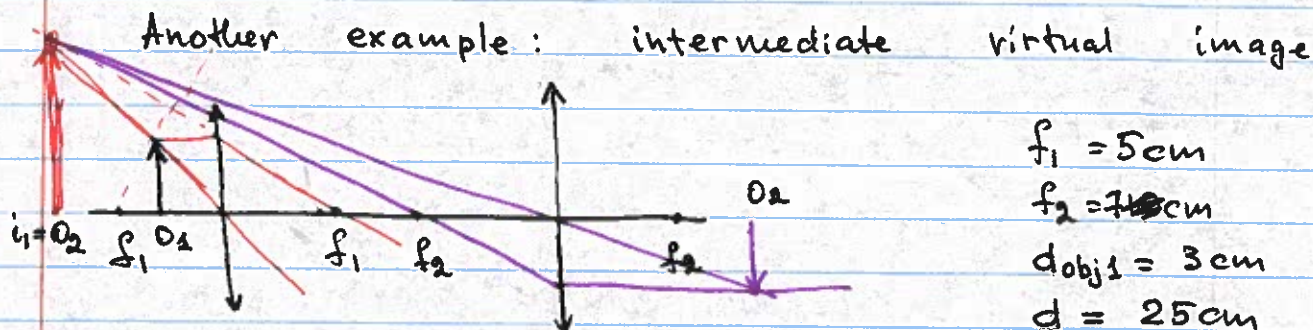
$$m_{\text{system}} = m_1 m_2 = \left(\frac{d_{\text{img1}}}{d_{\text{obj1}}} \right) \left(\frac{d_{\text{img2}}}{d_{\text{obj2}}} \right)$$

In our example

$$m_1 = - \left(\frac{11.25 \text{ cm}}{9 \text{ cm}} \right) = -\frac{5}{4} \quad m_2 = - \left(\frac{14.26 \text{ cm}}{19.75 \text{ cm}} \right)$$

$$m_{\text{syst}} \approx 1.3$$

real, erect
final image



$$d_{\text{img1}} = \frac{d_{\text{obj1}} \cdot f_1}{d_{\text{obj1}} - f_1} = -7.5 \text{ cm}$$

$$m_1 = - \frac{d_{\text{img1}}}{d_{\text{obj1}}} = 2.5$$

$$d_{\text{obj2}} = d + d_{\text{img1}} = 32.5 \text{ cm}$$

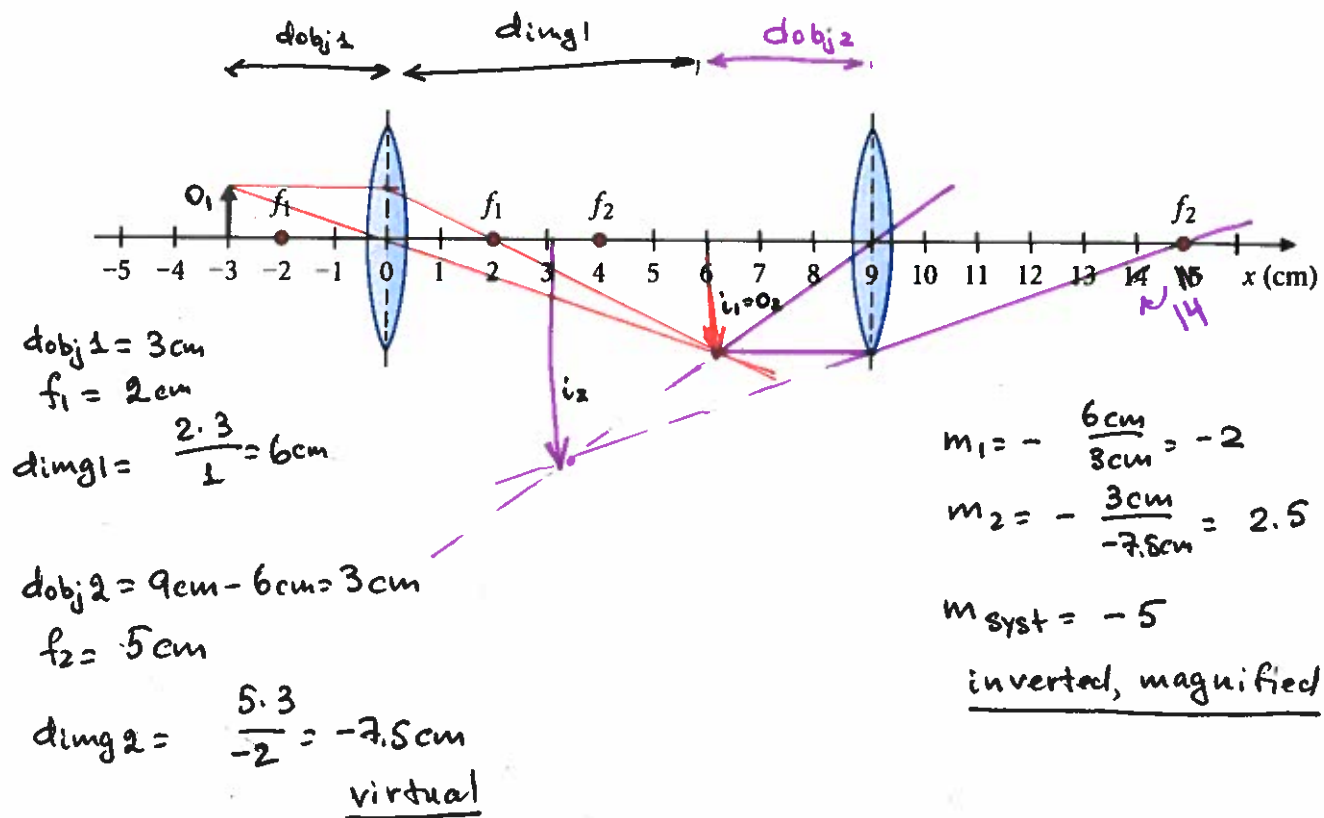
$$d_{\text{img2}} = \frac{d_{\text{obj2}} \cdot f_2}{d_{\text{obj2}} - f_2} = 8.92 \text{ cm}$$

$$m_2 = - \frac{d_{\text{img2}}}{d_{\text{obj2}}} = -0.27$$

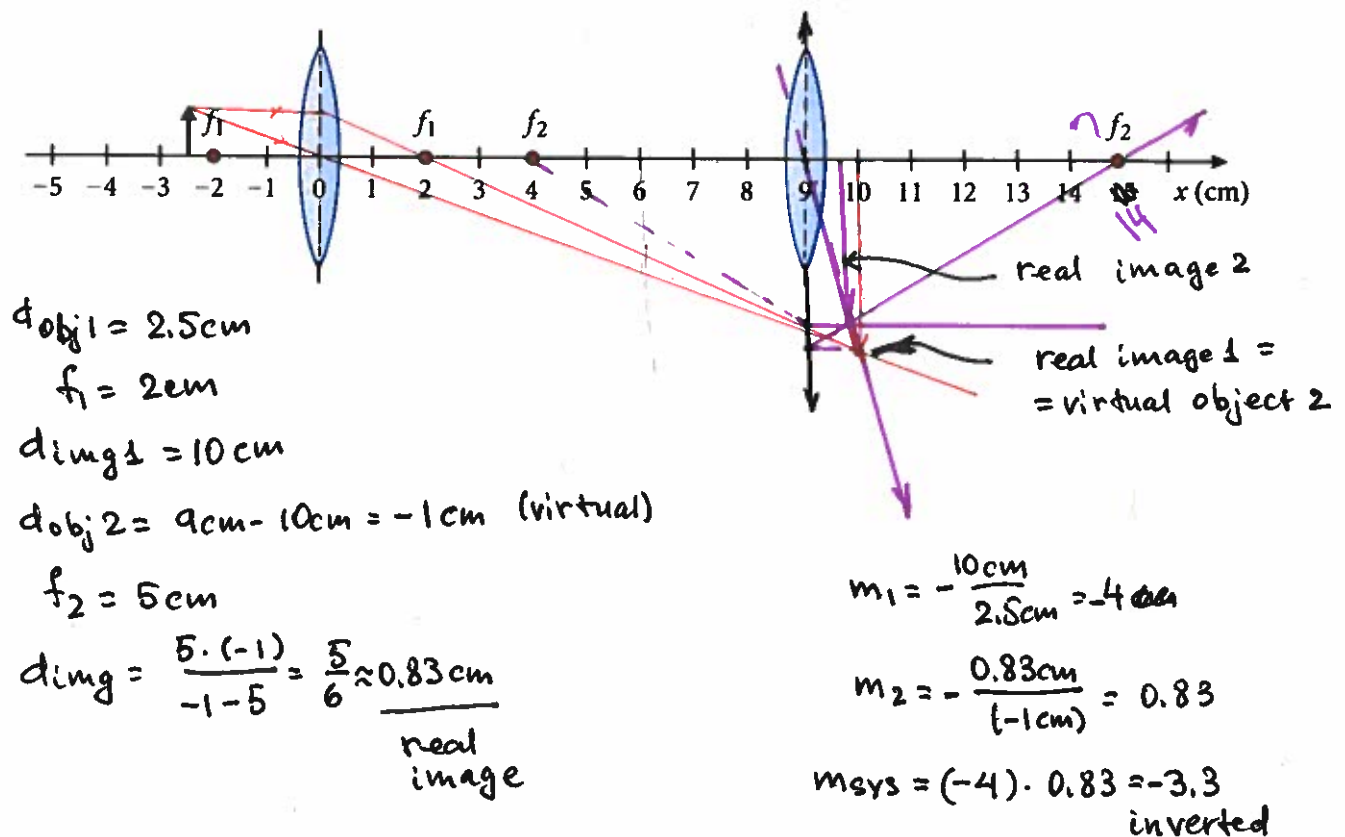
$$m_{\text{syst}} = m_1 \cdot m_2 = -0.68$$

real, inverted object

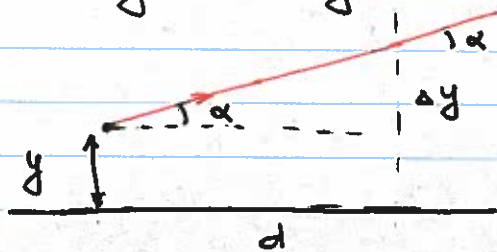
More straightforward (everyone should do this one)



More challenging (if you can do that one, you really mastered image formation)



Ray tracing - ABCD matrix method



at any point along the optical axis we can define the ray with its angle (in radians) and vertical position

If a ray traveled distance d along the axis d is unchanged

$$y \rightarrow y + \Delta y = y + d \cdot \tan \alpha \approx y + d \cdot \alpha$$

We assume small angles $\sin \alpha \approx \tan \alpha \approx \alpha$

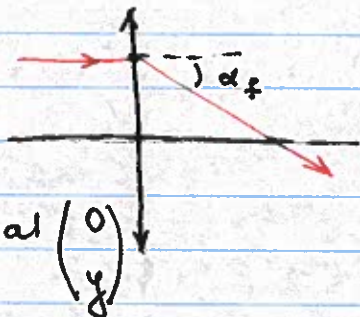
Initial ray parameters $\begin{pmatrix} d \\ y \end{pmatrix}$

Final ray parameters $\begin{pmatrix} d \\ y + d\alpha \end{pmatrix}$

$$\begin{pmatrix} d \\ y + d\alpha \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ d & 1 \end{pmatrix} \begin{pmatrix} d \\ y \end{pmatrix}$$

displacement/translation matrix

$$S_T = \begin{pmatrix} 1 & 0 \\ d & 1 \end{pmatrix}$$



$$\alpha_f \approx -\frac{y}{f}$$

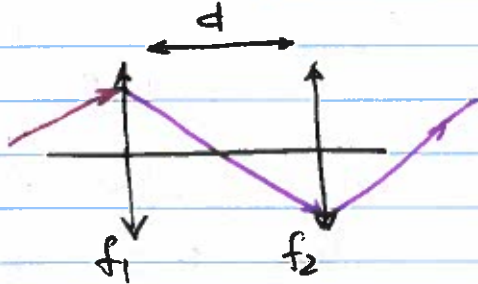
Initial $\begin{pmatrix} 0 \\ y \end{pmatrix}$

Final $\begin{pmatrix} -y/f \\ y \end{pmatrix}$

$$S_L = \begin{pmatrix} 1 & -1/f \\ 0 & 1 \end{pmatrix}$$

Each step of a ray propagation can be assign its 2×2 matrix (see wikipedia for examples under "Ray transfer matrix analysis")

Example: a system of two lenses



Initial ray: immediately before lens 1 $\begin{pmatrix} d_1 \\ y_1 \end{pmatrix}$

Transformation steps: lens 1 $\rightarrow$ translation (travels b/w the lenses) $\rightarrow$ lens 2

$$\begin{pmatrix} d_{fin} \\ y_{fin} \end{pmatrix} = S_{L_2} S_T S_{L_1} \begin{pmatrix} d_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 1 & -1/f_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ d & 1 \end{pmatrix} \begin{pmatrix} 1 & -1/f_1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 - \frac{d}{f_1 f_2} & -\frac{1}{f_1} - \frac{1}{f_2} \\ -\frac{d}{f_1} + 1 & 1 \end{pmatrix}$$

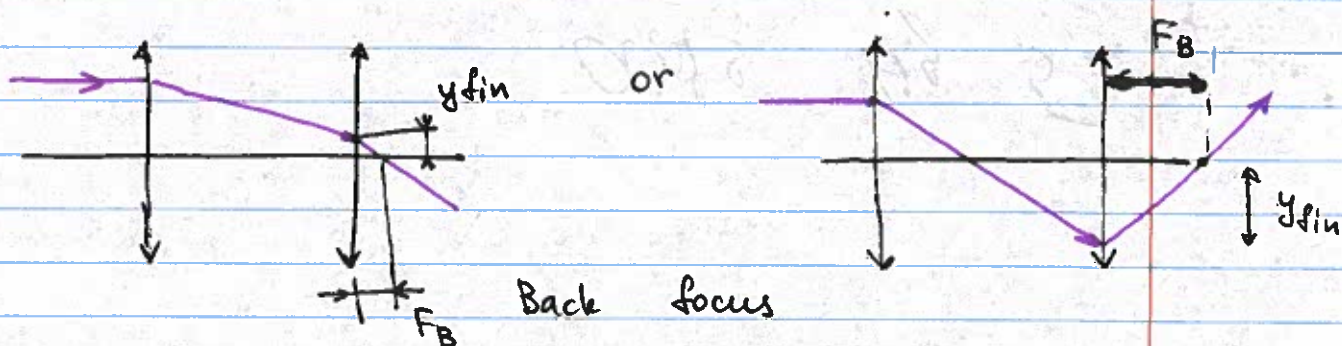
We can use this to find an effective focal length of a two-lens combination. If we start with $\begin{pmatrix} 0 \\ y \end{pmatrix}$ after the system

of lenses the beam will be going through its effective focus, f_{eff} and its angle $d_{fin} = -y/f_{eff}$

$$\begin{pmatrix} d_{fin} \\ y_{fin} \end{pmatrix} = \begin{pmatrix} 1 - \frac{d}{f_1 f_2} & -\frac{1}{f_1} - \frac{1}{f_2} \\ -\frac{d}{f_1} + 1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ y \end{pmatrix} = \begin{pmatrix} 1 - \frac{d}{f_1 f_2} & -\frac{1}{f_1} - \frac{1}{f_2} \\ -\frac{d}{f_1} + 1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ y \end{pmatrix} = \begin{pmatrix} 1 - \frac{d}{f_1 f_2} & -\frac{1}{f_1} - \frac{1}{f_2} \\ -\frac{d}{f_1} + 1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ y \end{pmatrix} = \begin{pmatrix} -\frac{y}{f_1} - \frac{1}{f_2} \left(-\frac{y d}{f_1} + y \right) \\ -\frac{y d}{f_1} + y \end{pmatrix}$$

$$\alpha_{fin} = \underbrace{\left[-\frac{1}{f_1} - \frac{1}{f_2} \left(-\frac{d}{f_1} + 1 \right) \right]}_{-1/f_{eff}} y$$

$$\frac{1}{f_{eff}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$



$$F_B = -\frac{y_{fin}}{\alpha_{fin}} = y_{fin} \cdot \frac{f_{eff}}{y} = \left(-\frac{d}{f_1} + 1 \right) \cdot f_{eff}$$