

Interferometers

are the workhorses of precision measurement.

Since changing an optical path by $\lambda/2$ flips bright interference fringe to a dark one (and vice versa), interference is ideal for tracking small changes in distance or refractive index, or wavelength.

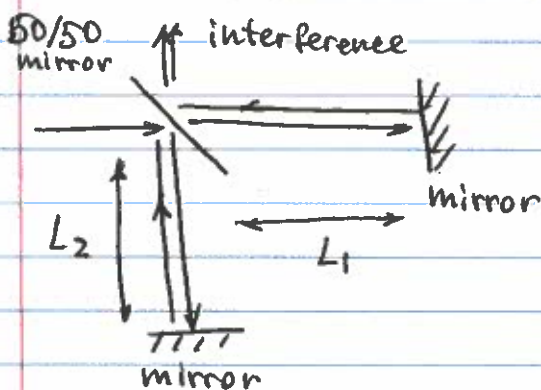
$$\Delta\phi = k\Delta x = \frac{2\pi}{\lambda} \Delta x$$

Two-slit interference is a very crude interferometer, with very few controls and a lot of wasted light.

In normal interferometers a light (usually the laser beam) is divided in two beams that travel through different paths, and then brought together to interfere.

There are many interferometer configurations depending on the intended measurement.

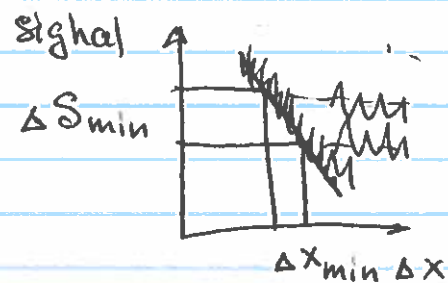
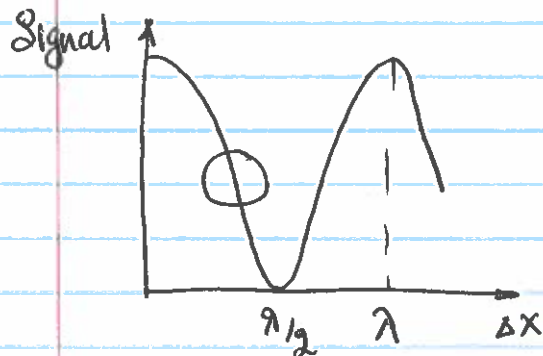
Michelson interferometer



if $(L_1 - L_2)$ changes by $\lambda/2$ (i.e. if one mirror shifts by $\lambda/4$) the fringe changes from dark to light.

So in principle, one can resolve tiny displacements by monitoring intensity changes in the "dark" ports.

The sensitivity of an interferometer (or any other sensor) is determined by how small change in the signal you can reliably measure.



Because of the presence of various noises, there will be a limit on how small variation in the ~~the~~ displacement or phase change one can measure.

If $\Delta\varphi_{\min}$ is the smallest phase variation we can resolve ~~at~~ with a particular interferometer, then for ~~the~~ the minimum measurable mirror displacement is

$$\Delta\varphi_{\min} = 2 \cdot \frac{2\pi}{\lambda} \cdot \Delta x_{\min} \Rightarrow \Delta x_{\min} = \lambda \cdot \frac{\Delta\varphi_{\min}}{4\pi}$$

$$\text{if } \Delta\varphi_{\min} = 10^{-3} \pi$$

$$\Delta x_{\min} = 10^{-3} \cdot \frac{\lambda}{4} \approx 2 \text{ nm for } \lambda = 800 \text{ nm (red)}$$

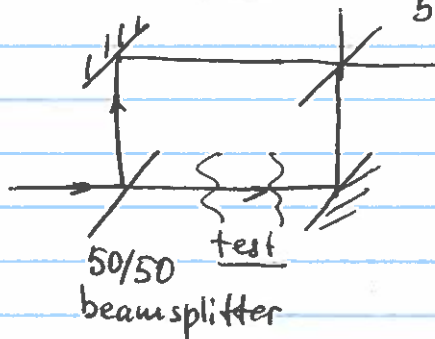
Michelson invented his interferometer to measure aether - a fictional (then) medium in which light assumed to be propagating. Having two arms perpendicular meant that speed of light should be different, creating changes in the interference depending on presumed aether motion.

Current most famous family of Michelson interferometers: LIGO (VIRGO, ...)
Light Interferometer Gravitational Observatory

Each arm - 4km (effectively 1000km with extra mirrors)

Mirror displacement by a gravitational wave
 $\frac{\Delta L}{L} \sim 10^{-23}$ or $\Delta L \sim 10^{-17} \text{ m} \sim 10^{-7}$ atom size

Mach-Zehnder interferometer
 50/50 beam splitter

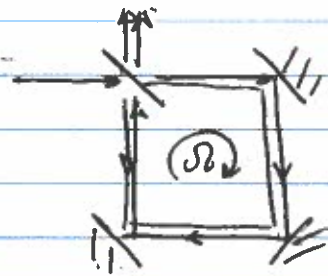


This configuration is perfect for quantum sensing, since a test object / phase shifter can be placed in one of the arms

Also commonly used in quantum information to manipulate quantum states of single photons

How to measure rotation?

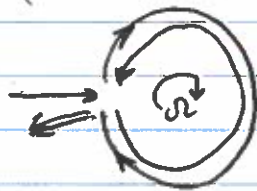
Sagnac Interferometer (basis for optical and laser gyroscopes)



Each of two beams travels around the circle in each direction

If the whole construction rotates with angular frequency Ω , during one round trip, so for one beam the ~~star~~ round trip is a little longer, and for the other - shorter

For simplicity, let's make cavity round



One period $T = \frac{2\pi R}{c}$

Beam path change

$$\Delta x_{\pm} = \pm \Delta \theta \cdot R = \pm \Omega \cdot T \cdot R$$

$$= \pm \Omega \cdot \frac{2\pi R^2}{c} = 2 \cdot A \cdot \Omega \cdot \frac{1}{c}$$

The phase difference b/w two beams

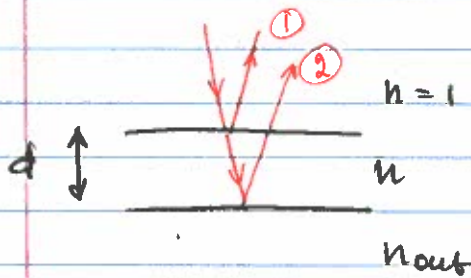
$$\Delta \varphi = k \Delta x_{+} - k \Delta x_{-} = 2 \cdot \frac{2\pi}{\lambda} |\Delta x_{\pm}| = \frac{8\pi}{\lambda c} \cdot A \cdot \Omega$$

If our sensitivity is limited by $\Delta \varphi_{\min}$ the minimum rotation rate $\Omega_{\min}$

$$\Omega_{\min} = \frac{\Delta \varphi_{\min}}{\frac{8\pi}{\lambda c}} \cdot \frac{c \cdot \lambda}{A}$$

To maximize the are, most modern optical gyroscopes use many turns of optical fiber, $A \rightarrow N \cdot A$

Another approach to measure small thickness changes and analyze thin films



Thin film interference is observed in reflection as interference of two beams reflected at two sides of the film acquired phase

① Top reflection

$+\pi$ if $n > 1$ (air)

② Bottom reflection

$$(2d) \cdot k = \frac{2\pi n}{\lambda_0} \cdot 2d = \frac{4\pi n d}{\lambda_0}$$

(λ_0 - wavelength in vacuum)

(if $n_{out} > n$ add extra π)

Assuming air - film - air interface total phase difference

$$\Delta\varphi = \frac{4\pi n d}{\lambda_0} - \pi$$

Note if $d \ll \lambda_0$ ($d/\lambda_0 \ll 1$) $\Delta\varphi = -\pi$
destructive interference, no reflection

Constructive interference: $\Delta\varphi = 2\pi \cdot m$

$$\frac{4\pi n d_m^{(max)}}{\lambda_0} - \pi = 2\pi m \quad m = 0, \pm 1, \pm 2$$

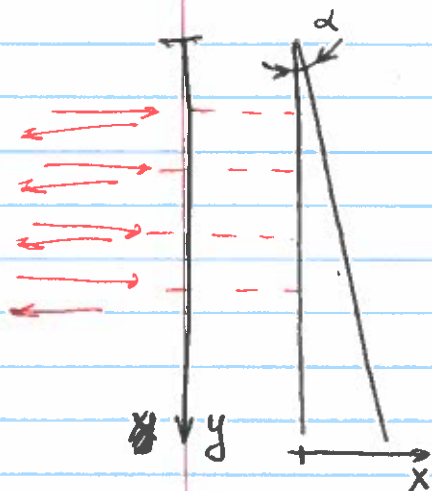
$$\frac{4\pi n d_m^{(max)}}{\lambda_0} = \pi + 2\pi m$$

$$4n \cdot d_m^{(max)} = (1 + 2m) \lambda_0$$

Destructive interference: $\Delta\varphi = -\pi + 2\pi m$

$$\frac{4\pi n d_m^{(min)}}{\lambda_0} - \pi = -\pi + 2\pi m \quad 4n d_m^{(min)} = 2m \lambda_0$$

For a given thickness different colors will be interfering differently



$$\frac{x}{y} = \tan \alpha \approx \alpha \text{ (in rad)}$$

$$y = x/d$$

for a given d and λ_0
when is the first bright reflection appear?

$$d_m^{(max)} = \frac{\lambda_0}{4n} (1 + 2m)$$

First band $m=0$

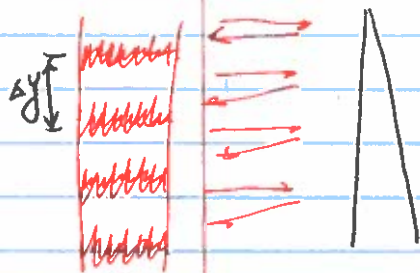
$$d_0^{(max)} = \frac{\lambda_0}{4n} \quad \text{or} \quad y = \frac{\lambda_0}{2nd}$$

We expect to see blue (smaller λ_0) first, then progress through the rainbow to the red

$$d_1^{(max)} = \frac{\lambda_0}{4n} \cdot 3$$

$$\Delta d = d_1 - d_0 = d_m - d_{m-1} = \lambda_0/2n$$

If using a single color, can measure small angles!



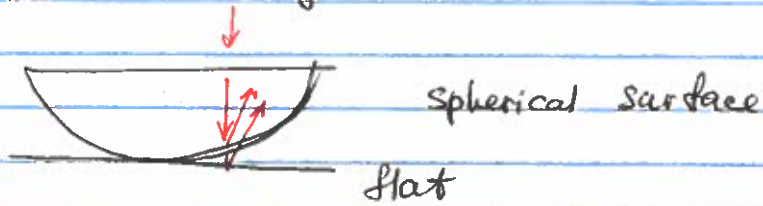
$$\Delta d = \lambda_0/2n$$

$$\Delta y = \frac{\Delta d}{\alpha} = \frac{\lambda_0}{2n\alpha}$$

$$\alpha = \frac{\lambda_0}{2n\Delta y}$$

In general, can determine the quality of optical surfaces

Newton rings



The interference pattern is formed by the reflections from the spherical and flat surfaces, the optical path difference is determined by the size of the air gap



r - distance from the touch point

R - radius of the spherical surface

one can show that

$$h = \frac{r^2}{2R}$$

Touch point - destructive interference (because of the π shift of one of the ~~two~~ reflected beams)

First constructive interference $2h_1 = \frac{\lambda}{2} = \frac{r_1^2}{2R}$
(bright ring)

Second bright ring $2h_2 = \frac{3\lambda}{2}$
 $r_2 = \sqrt{\frac{3}{2}\lambda R}$

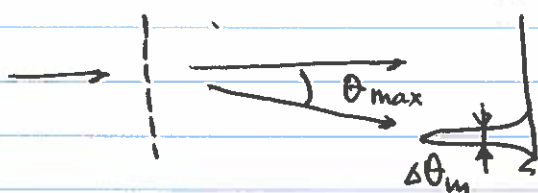
In general
$$r_m = \sqrt{\left(m + \frac{1}{2}\right) \lambda R}$$

Spectral resolution $\rightarrow$ how to separate different colors

1. Classic - use material dispersion in a prism
Sufficient only to separate colors with distinctly different wavelengths



2. Diffraction grating



$$\theta_{\max}^{(m)} = m \frac{\lambda}{d}$$

The width of the maxima (angle to the closest min)

$$\Delta\theta_m = \frac{\lambda}{N \cdot d}$$

N - number of participating slits

$$\Delta\theta_{\Delta\lambda} = m \frac{\lambda + \Delta\lambda}{d} - m \frac{\lambda}{d} = m \frac{\Delta\lambda}{d} \geq \frac{\lambda}{N \cdot d}$$

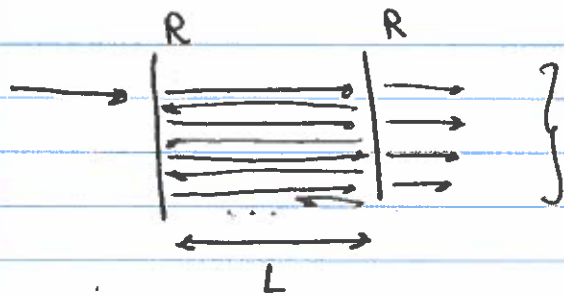
$$\Delta\lambda_{\min} \sim \lambda / m \cdot N$$

A standard diffraction grating can have 500-600 grooves/m. If a laser beam is 2mm diameter $N \sim 1000$, $m = 2, 3$

$$\Delta\lambda_{\min} \sim \frac{\lambda}{2 \cdot 1000} \approx 5 \cdot 10^{-4} \lambda$$

$$\text{if } \lambda = 500\text{nm} \quad \Delta\lambda = 0.25\text{nm}$$

3. Fabri- Perot interferometer (multi-beam interferometer)



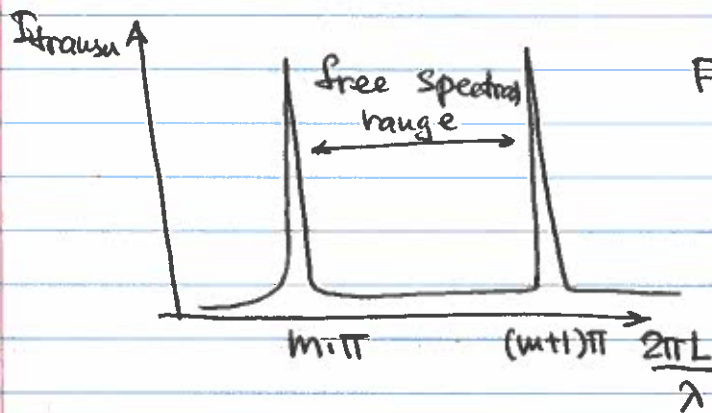
All transmissions interfere

$$I_{\text{transm}} = I_0 \frac{1-R}{1-R \cos^2\left(\frac{2\pi L}{\lambda}\right)}$$

R - reflection coefficient of a mirror ($R \sim 1$)

If $(1-R) \ll 1$ then for most L $I_{\text{transm}} \sim 0$

However if $\cos^2\left(\frac{2\pi L}{\lambda}\right) = 1$ ($\frac{2\pi L}{\lambda} = \pi \cdot m$)
 $I_{\text{transm}} = I_0$ all light goes through!



Finesse = $\frac{\text{distance b/w peaks}}{\text{width of a peak}}$

$$= \pi \frac{\sqrt{R}}{1-R} \approx \frac{\pi}{1-R}$$

The closer R to 1, the higher is the cavity finesse

~~Reso~~ Spectral resolution

$$\frac{\lambda}{\delta\lambda} = \frac{\pi \sqrt{R}}{1-R} = 10^3 - 10^6 \text{ depending on mirror quality}$$

up to $10^8 - 10^9$ in special configurations