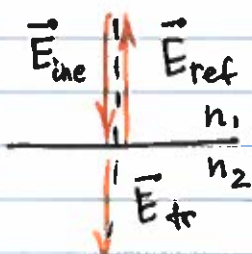


Interaction of light with materials (and why polarization matters)

When we talked about reflection or refraction, we established the rules regarding the directions of reflected and refracted beams, but didn't discuss how much of incoming light is reflected.

For example, if I took at reflection only, can I distinguish if the light is reflected from water ($n_{\text{water}} = 1.33$), glass ($n_{\text{glass}} = 1.5$) or diamond ($n_{\text{dia}} = 2.3$)? The angle of reflection will be the same in all three cases, but the amount of light reflected will differ.

For a moment we'll consider normal incidence $\theta_{\text{inc}} = \theta_{\text{ref}} = 0$



Reflection coefficient

$$|\vec{E}_{\text{ref}}| = r |\vec{E}_{\text{inc}}|$$

For normal incidence

$$r = \frac{n_1 - n_2}{n_1 + n_2}$$

If $n_1 \approx 1$ (air)

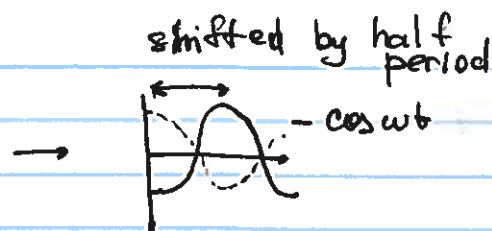
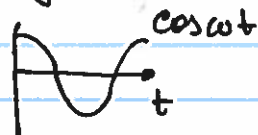
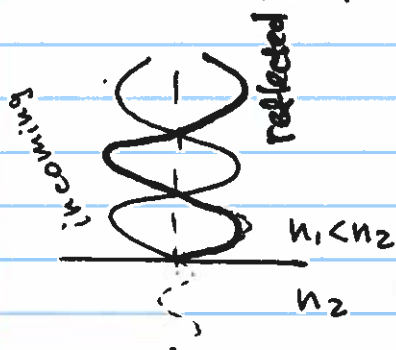
$$r_{\text{water}} = \frac{1 - 1.33}{1 + 1.33} = \frac{-0.33}{2.33} \approx -0.14$$

$$r_{\text{glass}} = \frac{1 - 1.5}{1 + 1.5} = -0.2$$

$$r_{\text{dia}} = -0.39$$

Why r is negative?

Quick reminder



Negative sign on r means the wave is shifted by half a wavelength (or by phase $\pi = 180^\circ$)

How bright is the reflected beam? Brightness = intensity

$$I = \frac{c\epsilon_0}{2} |E|^2$$

$$I_{inc} = \frac{c\epsilon_0}{2} |E_{inc}|^2$$

$$I_{ref} = \frac{c\epsilon_0}{2} |E_{ref}|^2 = \frac{c\epsilon_0}{2} |r E_{inc}|^2$$

$$= \frac{c\epsilon_0}{2} |r|^2 |E_{inc}|^2 = |r|^2 I_{inc}$$

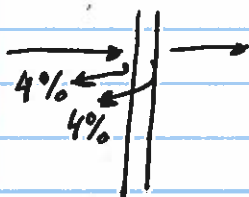
Reflection coefficient for intensity (power)

$$R = |r|^2 = \frac{I_{ref}}{I_{inc}} = \left(\frac{n_1 - n_2}{n_1 + n_2} \right)^2$$

$$R_{water} = (-0.14)^2 = 0.02 \text{ (2\%)} \quad R_{glass} = (-0.2)^2 = 0.04 \text{ (4\%)}$$

$$R_{dia} = (1 - 0.39)^2 = 0.155 \text{ (15.5\%)}$$

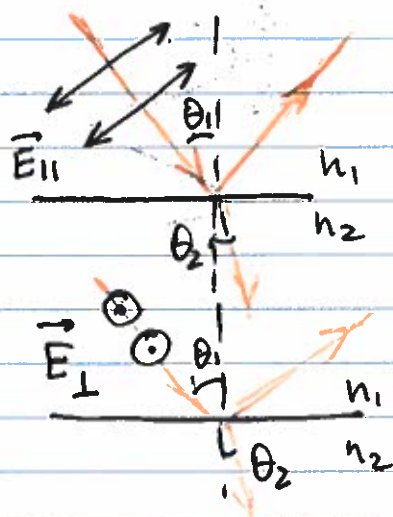
Quick check: how much light is reflected of a glass window?



Since window has two glass-air surfaces, each will reflect 4%, so total reflection is 8%

Transmission coefficient for intensity $T = 1 - R = \frac{4n_1 n_2}{(n_1 + n_2)^2}$

If the light ray ~~is~~ arrives at non-zero incident angle, the reflection is different for different polarizations



Parallel polarization
($\Rightarrow$ parallel to the incidence plane)
P-polarization

Perpendicular polarization
($\Rightarrow$ perpendicular to the incidence plane)
S-polarized

For parallel polarization reflection coeff:

$$r_{||} = \frac{E_{\text{refl}}}{E_{\text{inc}}} = \frac{n_2 \cos \theta_1 - n_1 \cos \theta_2}{n_2 \cos \theta_1 + n_1 \cos \theta_2}$$

(r_p)

transmission coeff:

$$t_{||} = \frac{E_{\text{tr}}}{E_{\text{inc}}} = \frac{2n_1 \cos \theta_1}{n_2 \cos \theta_1 + n_1 \cos \theta_2}$$

(t_p)

For perpendicular polarization

$$r_{\perp} = \frac{n_1 \cos \theta_1 - n_2 \cos \theta_2}{n_1 \cos \theta_1 + n_2 \cos \theta_2}$$

(r_s)

$$t_{\perp} = \frac{2n_1 \cos \theta_1}{n_1 \cos \theta_1 + n_2 \cos \theta_2}$$

(t_s)

Since any polarization can be presented as a combination of two orthogonally polarized waves, if we know ~~the~~ these coefficients, we can calculate reflection and transmission for any polarization

-4-

Reflection coefficient for Intensity

$$R = \frac{I_{\text{ref}}}{I_{\text{inc}}} = |r|^2$$

Transmission coefficient for intensity

$$T = \frac{I_{\text{tr}}}{I_{\text{inc}}} = \frac{n_2}{n_1} \left(\frac{\cos \theta_2}{\cos \theta_1} \right) |t|^2 \quad \text{to account for the speed of light difference}$$

(or $T = 1 - R$ because of the energy conservation)

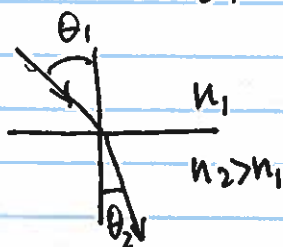
Now let's look back on the reflection and transmission coefficients.

$t_{\parallel}$ and $t_{\perp} > 0$ for any $\theta_1 < 90^\circ$

However, there are differences in the numerators for both. Can one of them go to zero (i.e. no reflection?)

$$r_{\parallel}: n_2 \cos \theta_1 - n_1 \cos \theta_2 \stackrel{?}{=} 0 \quad r_{\perp}: n_1 \cos \theta_1 - n_2 \cos \theta_2 \stackrel{?}{=} 0$$

Plus: $n_1 \sin \theta_1 = n_2 \sin \theta_2$ Snell's law



if $n_1 < n_2$ then $\theta_1 > \theta_2$ and $\cos \theta_1 < \cos \theta_2$

$r_{\perp}$: So $n_1 \cos \theta_1 < n_2 \cos \theta_2$ for any θ_1

But! One can find an angle

for which $n_2 \cos \theta_1 = n_1 \cos \theta_2$, so $r_{\parallel} = 0$

Brewster angle $\theta_{\text{Br}}: \tan \theta_{\text{Br}} = \frac{n_2}{n_1}$

Air-water interface: $n_2 = 1.33$ $\tan \theta_{\text{Br}} = 1.33$

$$\theta_{\text{Br}} = 53^\circ$$

- 4 extra -

For extra fun: derivation of $\tan \theta_{Br} = n_2/n_1$
Only if you like math.

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$n_2 \cos \theta_1 = n_1 \cos \theta_2$$

$$\frac{n_1}{n_2} \sin \theta_1 = \sin \theta_2$$

$$\frac{n_2}{n_1} \cos \theta_1 = \cos \theta_2$$

square
each
and add

$$1 = \sin^2 \theta_2 + \cos^2 \theta_2 = \left(\frac{n_1}{n_2}\right)^2 \sin^2 \theta_1 + \left(\frac{n_2}{n_1}\right)^2 \cos^2 \theta_1$$

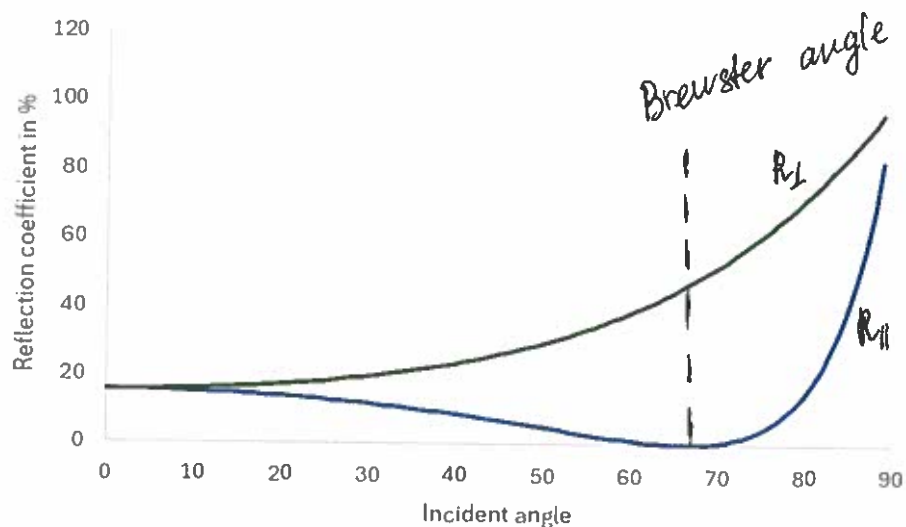
$$\sin^2 \theta_1 + \cos^2 \theta_1 = \left(\frac{n_1}{n_2}\right)^2 \sin^2 \theta_1 + \left(\frac{n_2}{n_1}\right)^2 \cos^2 \theta_1$$

$$\left(\frac{n_1}{n_2}\right)^2 \sin^2 \theta_1 - \sin^2 \theta_1 = \cos^2 \theta_1 - \left(\frac{n_2}{n_1}\right)^2 \cos^2 \theta_1$$

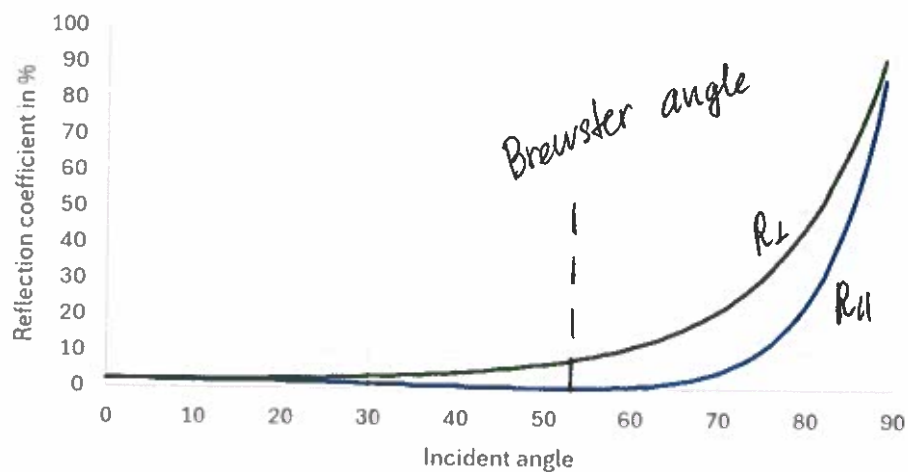
$$\frac{(n_1^2 - n_2^2)}{n_2^2} \sin^2 \theta_1 = \frac{(n_1^2 - n_2^2)}{n_1^2} \cos^2 \theta_1$$

$$\frac{\sin^2 \theta_1}{\cos^2 \theta_1} = \tan^2 \theta_1 = \frac{n_2^2}{n_1^2} \Rightarrow \tan \theta_{Br} = \frac{n_2}{n_1}$$

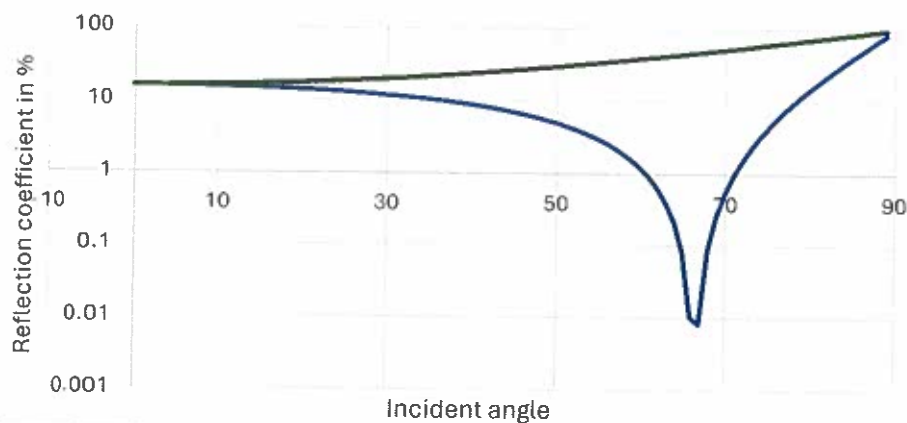
(Intensity) reflection coefficient for diamond ($n=2.3$)



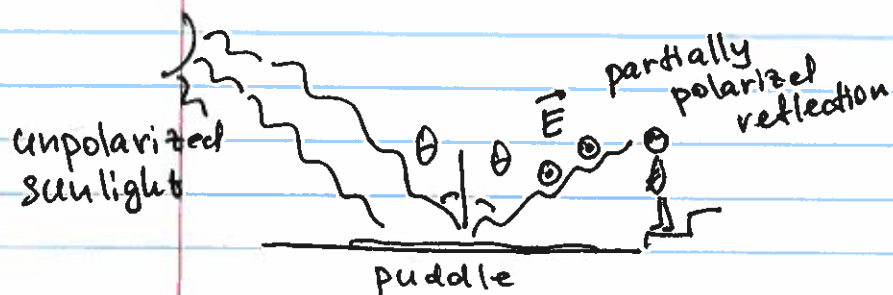
(Intensity) reflection coefficient for water ($n=1.33$)



(Intensity) reflection coefficient for ~~water~~ diamond
($n=2.3$)



So why polarization glasses make sense?



Even though the sunlight is not polarized, if it is reflected at angles close to the Brewster angle, the reflected light is partially polarized, and thus this glare can be blocked (~~per~~ reduced) using polarized that block perpendicular polarization. In a normal geometry perpendicular = horizontal, so a proper polarizing sunglasses should block horizontal polarization.