Single particle quantum state

Let us assume that some particle (an electron) can be in only two distinguishable quantum states:

Electron spin \( S = \frac{1}{2} \), \( S_z = \pm \frac{1}{2} \)

Two states: spin up \( \uparrow \) (\( S_z = \frac{1}{2} \)) and spin down \( \downarrow \) (\( S_z = -\frac{1}{2} \))

In principle, it can be any superposition of these two states:

\[ \psi = A \psi_\uparrow + B \psi_\downarrow = A \uparrow + B \downarrow \]

Measurement collapses the wave function!

\( A \uparrow + B \downarrow \) (Measurement) \( \Rightarrow \) \( \uparrow \) with \( P = |A|^2 \)

\( A \uparrow + B \downarrow \) \( \Rightarrow \downarrow \) with \( P = |B|^2 \)

After the measurement, the state changes.

Two independent particles

\[ \psi_1 = A_1 \uparrow + B_1 \downarrow \quad \psi_2 = A_2 \uparrow + B_2 \downarrow \]

\[ P_{\psi_1\psi_2} = P_{\psi_1} \cdot P_{\psi_2} = |A_1|^2 |B_2|^2 \]

Two-particle wave function \( \psi_{12} = \psi_1 \cdot \psi_2 = (A_1 \uparrow + B_1 \downarrow)(A_2 \uparrow + B_2 \downarrow) \)

Measurement of one particle does not tell anything about the other.
Correlated particles

\[ S_z^1 \quad \circlearrowleft \quad \circ \quad \circlearrowright \quad S_z^2 \]

Total spin must remain zero, so two possibilities:

- \( S_{1z} = \frac{1}{2} \quad S_{2z} = -\frac{1}{2} \quad \left| \uparrow \right>_1 \left| \downarrow \right>_2 \)
- \( S_{1z} = -\frac{1}{2} \quad S_{2z} = \frac{1}{2} \quad \left| \downarrow \right>_1 \left| \uparrow \right>_2 \)

Two particle wave function:

\[ \psi_{1,2} = \frac{1}{\sqrt{2}} \left( \left| \uparrow \right>_1 \left| \uparrow \right>_2 - \left| \uparrow \right>_1 \left| \downarrow \right>_2 \right) \]

Such two particles are entangled:

Knowing something about one of them provides information about the other!

That gives rise to an EPR paradox.

For example:

Quantum mechanics does not allow knowing two components of angular momentum at the same time.

One can measure \( S_z \) or \( S_x \), but not both!

Entangled particles

Stern-Gerlach apparatus (SGA) measures z-component (or x-component) depending on orientation.
\[ \Psi_2 = \frac{1}{\sqrt{2}} (|\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2) \] along $z$-direction

at the same time

\[ \Psi_2 = \frac{1}{\sqrt{2}} (|\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2) \] along $x$-direction

\( \mathbf{S}_z \): 
- \( S_{z1} = +\frac{1}{2} \), \( |\uparrow\rangle \)
- \( S_{z2} = -\frac{1}{2} \), \( |\downarrow\rangle \)

\( \mathbf{S}_x \): 
- \( S_{x1} = +\frac{1}{2} \)
- \( S_{x2} = -\frac{1}{2} \)

Particle 1: \( S_z = \frac{1}{2} \), \( S_x = \frac{1}{2} \) is QM wrong?! 

No

First measurement collapses the wave function

\[ \Psi_2 \rightarrow (|\uparrow\rangle_1 |\downarrow\rangle_2) \]

but \( |\downarrow\rangle_2 = \frac{1}{\sqrt{2}} (|\uparrow\rangle_2 - |\downarrow\rangle_2) \)

If you measure $S_x$ now, you lose information about $S_z$

\[
\text{angle b/w detectors}
\]

\[
\text{angle b/w detectors}
\]
Bell's inequality

Locality - each particle has to carry all the information with it, even if it is not measured.

\[ x = \pm \frac{1}{2}, \quad y = \pm \frac{1}{2}, \quad z = \pm \frac{1}{2} \]

\[ a \quad b \quad c \]

\[ P_a \rightarrow S_x = \pm \frac{1}{2}, \quad P_b \rightarrow S_y = \pm \frac{1}{2}, \quad P_{ac} \rightarrow S_{x_1} = \frac{1}{2}, \quad S_{y_1} = \frac{1}{2} \]

\[ P_{ab} + P_{bc} \geq P_{ac} \]

If Bell's inequality is true - the system can be described by local variables; violation of Bell's inequality means QM correlation.