Relativistic Doppler effect

Stationary source
\[ v = 0 \]
\[ f_0 = \frac{1}{T_0} \]

Moving source
\[ t = 0 \quad v \quad t = \delta T_0 \]

Let's assume that when the source was lined up with the stationary detector, it saw the crest of the wave. When the detector sees the next one?

In the source's RF — after \( T_0 \).
In the detector's RF — after \( \delta T_0 \) (time dilation).

The source travelled by \( v = \delta T_0 \), so it takes light \( \delta T_0 / c \) to travel back to the detector.

Total time b/w to detected peaks:

\[ T_D = \delta T_0 + \delta T_0 / c = \delta T_0 (1 + \frac{v}{c}) = T_0 \left( \frac{1 + \frac{v}{c}}{\sqrt{(1 - \frac{v^2}{c^2})}} \right) \]

\[ T_D = T_0 \left( \frac{1 + \frac{v}{c}}{\sqrt{1 - \frac{v^2}{c^2}}} \right) \]

\[ f_D = \frac{1}{T_D} \]

\[ f_D = f_0 \sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}} \]
If the source is receding from the detector:

\[ f_D = f_0 \sqrt{\frac{1 - v/c}{1 + v/c}} < f_0 \quad \text{"red shift"} \]

If the source is approaching the detector:

\[ f_D = f_0 \sqrt{\frac{1 + v/c}{1 - v/c}} > f_0 \quad \text{"blue shift"} \]

Transverse Doppler effect:

\[ f_D = \frac{f_0}{\gamma(1 + \frac{v \cos \theta}{c})} \]

* No Doppler shift for sounds
* Light: Still have time dilation for the source

\[ T_D = \gamma T_0 \]

\[ f_D = \frac{1}{\gamma} f_0 = \sqrt{1 - \frac{v^2}{c^2}} f_0 \]

In general:

\[ f_D = \frac{f_0}{\gamma(1 + \frac{v \cos \theta}{c})} \]