Bohr's atomic model

\[ F_e = \frac{ke^2}{a^2} = \frac{ma^2}{a} \Rightarrow a = \frac{ke^2}{mv^2} \]

Recalling \( \omega = \sqrt[3]{a} \) angular frequency

\[ \frac{ke^2}{a^2} = m\omega^2 a \Rightarrow Rotation \ Frequency \]

\[ \omega = \sqrt[3]{\frac{ke^2}{ma^2}} \]

Higher frequency \( \Rightarrow \) smaller orbit

1. An electron in an atom moves in a circular orbit about the nucleus under the influence of the Coulomb attraction between the electron and the nucleus, and obeying the laws of classical mechanics.

2. But, instead of the infinity of orbits which would be possible in classical mechanics, it is only possible for an electron to move in an orbit for which its orbital angular momentum \( L \) is an integral multiple of Planck's constant \( h \), divided by \( 2\pi \).

3. Despite the fact that it is constantly accelerating, an electron moving in such an allowed orbit does not radiate electromagnetic energy. Thus its total energy \( E \) remains constant.

4. Electromagnetic radiation is emitted if an electron, initially moving in an orbit of total energy \( E_i \), discontinuously changes its motion so that it moves in an orbit of total energy \( E_f \). The frequency of the emitted radiation \( \nu \) is equal to the quantity \( (E_i - E_f) \) divided by Planck's constant \( h \).
Quantization of orbital motion

\[ L = \mathbf{r} \times \mathbf{p} \]

For a circular orbit

\[ L = ma \cdot v \quad \text{or} \quad L = ma^2 \omega \]

\[ L = n \cdot \frac{\hbar}{2\pi} = n \hbar = ma \cdot v \implies v = \frac{n \cdot \hbar}{ma} \]

\[ a = \frac{ke^2}{mv^2} = \frac{ke^2}{n^2} \cdot \frac{m^2 a^2}{n^2 \hbar^2} \implies a = \frac{n^2 \hbar^2}{ke^2 m} \]

Bohr's radius \( a_0 = \frac{\hbar^2}{ke^2 m} \approx 5 \times 10^{-10} \text{ m} \)

Electrons in Bohr's atom move along specific orbits, and radii of these orbits are

\[ a_n = n^2 a_0 = \frac{n^2 \hbar^2}{ke^2 m} \]

Electron's velocity \( v = \frac{n \hbar}{ma} = \frac{ke^2}{n \hbar} \approx 10^6 \text{ m/s} \)

\( v \ll c \) non-relativistic

Energy of an electron

\[ E = K + U = \frac{mv^2}{2} - \frac{ke^2}{a} \]

\[ K = \frac{mv^2}{2} = \frac{1}{2} \left[ \frac{mv^2}{a} \right] \cdot a = \frac{1}{2} \left[ \frac{ke^2}{a^2} \right] \cdot a = \frac{1}{2} \frac{ke^2}{a} \]

\[ E = -\frac{1}{2} \frac{ke^2}{a} = -\frac{me^4 k^2}{2n^2 \hbar^2} = -\frac{\hbar^2}{2me^2 a_0^2} \frac{1}{n^2} \]
Energy difference – photon frequency

\[ E_{\text{photon}} = \hbar \omega = E_{n_f} - E_{n_i} \]

\[ \hbar \omega = \frac{m e^4 k^2}{2 \hbar^2} \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \]

in excellent correspondence to Rydberg’s formula

\[ a_n = \hbar^2 a_0 \]

The hydrogen atom according to Bohr’s theory. Both are partial diagrams. The larger one illustrates some of the jumps between quantum orbits which give rise to the different frequencies in hydrogen’s spectrum. The three lines in its visible region, known as “the Balmer series,” are labeled H\(_a\), H\(_b\) and H\(_c\). The smaller diagram is a slightly different (and more familiar) representation of the Bohr hydrogen atom.
Franck-Hertz Data for Mercury

This original Franck-Hertz data shows electrons losing 4.9 eV per collision with mercury atoms. It is possible to observe ten sequential bumps at intervals of 4.9 volts.

Sketch of apparatus

Further discussion

1. accelerates
2. collides with Hg
3. reaccelerates after collision
4. energy transferred to Hg

→ accelerates to 4.9eV, collides with Hg and loses energy, reaccelerates to 4.9eV, loses energy in the second collision with Hg,

and so on...
Positively charged grid accelerates electrons.

Collecting plate is slightly negative with respect to the grid so that only those electrons above an energy threshold will reach it.

Heated cathode produces electrons.

Mercury vapor.

Current from collector measured as a function of accelerating voltage.

After Krane.

Without atoms.

Higher accelerating voltage → more electrons reach the collector plate.
Correspondence principle

Predictions of quantum theory must correspond to the predictions of classical physics in the region of parameters (or quantum numbers) where classical theory is known to hold.

Classical limit for an atom: large $n$ (orbits become so close to each other that they are almost continuous)

\[
\frac{\Delta E_n}{2} = \frac{k^2 e^2}{r_n^2} \frac{\Delta r_n}{r_n^2} = \frac{1}{2} m \omega^2 r_n \Delta r_n
\]

for large $n$ \( \Delta r_n \ll r_n \)

Angular momentum \( L = m \omega r^2 \)

\[\Delta L = 2m \omega r \Delta r + mr^2 \Delta \omega\]

\( r \) and \( \omega \) are not independent. \( m \omega r = \frac{k^2 e^2}{r^2} \)

\( \Rightarrow \omega = \frac{k^2 e^2}{m r^3} \Rightarrow 2\omega \Delta \omega = \frac{3k^2 e^2}{m r^4} = -3 \frac{e}{r} \Delta r \)

\[\Delta L = 2m \omega r \Delta r - \frac{3}{2} m \omega r \Delta r = \frac{1}{2} m \omega r \Delta r\]

Thus \( \Delta E_n = \omega \Delta L = \hbar \omega_{EH} = h \frac{f_{EH}}{\Delta r} \)

For rotating electron \( e \omega = \omega \)

\( \Delta L = \hbar \)